ONLINE WORKSHOP ON "COMPUTATIONAL KNOT THEORY"

`http://www.stoimenov.net/stoimeno/homepage/ckt`

organizers: Alexander Stoimenow, Hyunwoo Lee

hosted by Prof Martin Ziegler's lab at

KAIST, School of Computing

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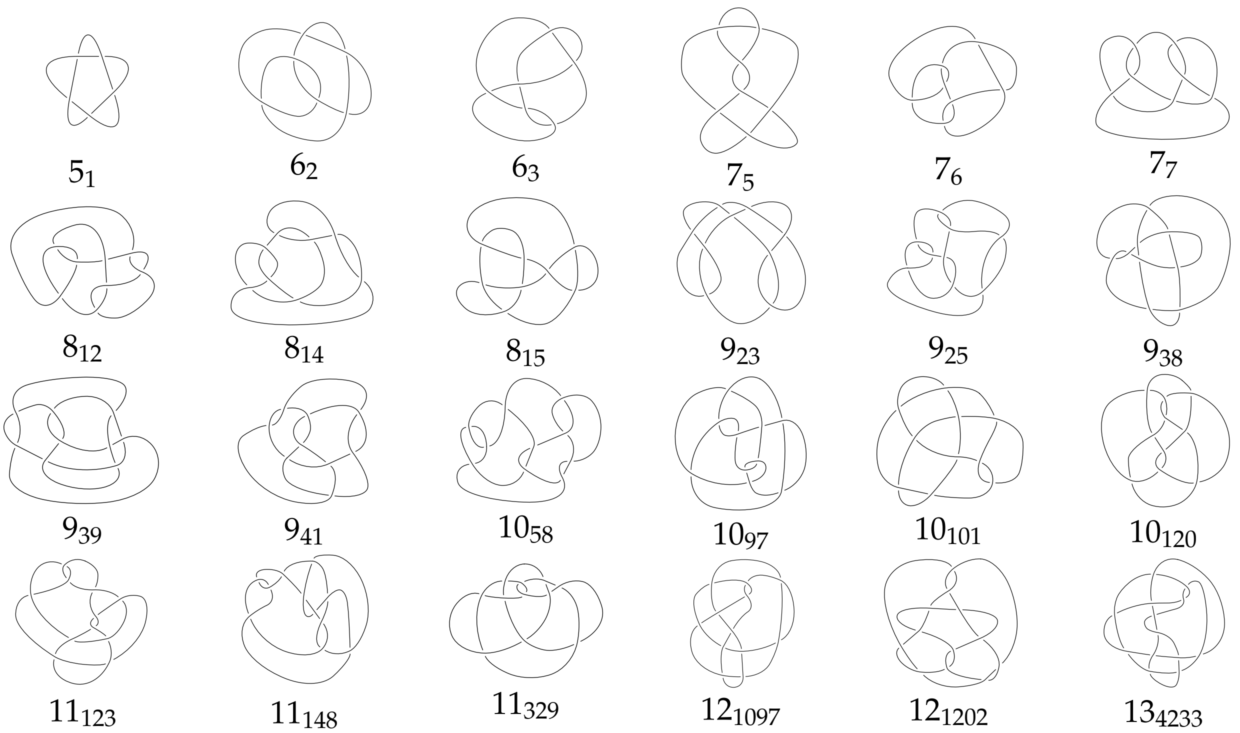
Program



Our goal is also to promote the topic with talks accessible to (and whet interest of) computer science students (who have not heard much about knot theory).

zoom ID **875 1903 7390**, password **"knotty"**.

Talk time should be **25-50 minutes** (+ possible questions). **Times are KST** (= GMT +0900).

For a *very* basic introduction, students may consult notes and links to my previous talks at the dept, "Introduction to Computational Knot Theory", "Knotscape and knot tables" (see the workshop page).

Colin Adams (Williams College) 5/26 10am

Multi-crossing Number and Petal Number for Knots

Instead of considering pictures of knots with two strands crossing at every crossing, we can ask for n strands to cross at every crossing. We will show that every knot and link has such an n-crossing projection for all integers n greater than 1 and therefore an n-crossing number. We also show that every knot has a projection with a single such multicrossing and no nested loops, resembling a daisy. This a petal projection which generates a petal number. We will discuss work done by undergraduates on determining these numbers, and mention a variety of open problems.

Stepan Orevkov (Universite P. Sabatier Toulouse) 5/26 8pm

Computation of multivariable signatures of colored links

A colored link is a link with a fixed partition of its connected components into several sublinks (which we call monochrome components). Multivariable signatures of were introduced and studied by Oleg Viro and Vincent Florens. They serve as an upper bound for the Euler characteristic of a slice surface whose connected components have monochrome boundaries.

David Cimasoni and Vincent Florens computed the multivariable signatures via the signatures of a generalized Seifert form on the 1-st homologies of a C-complex of a colored link. In my talk I discuss the computational problems appearing in this context.

Gyo Taek Jin (KAIST) 5/27 10am

Arc index and minimal grid diagrams of prime knots

Every knot in space can be embedded in a finitely many half planes attached to the z-axis in such a way that each half plane contains a single arc whose ends are on the z-axis. Such embeddings are called arc presentations. The minimal number of such arcs for all possible arc presentations of a knot is called its arc index. A grid diagram is a knot diagram with finitely many horizontal line segments and the same number of vertical line segments such that at each crossing the vertical segment goes over the horizontal segment. Grid diagrams are easily converted to arc presentations and vice versa. We describe how the arc index of prime knots up to certain ranges are obtained. We also describe how minimal grid diagrams of the 11 crossing prime alternating knots and the 12 crossing prime alternating knots are obtained.

Roland van der Veen (University of Groningen) 5/27 8pm

OU tangles and effective universal knot invariants

Studying knots through planar projections (knot diagrams) is challenging because many diagrams represent the same knot. Is there some way of selecting a preferred diagram for a knot? We attempt to bring knot diagrams in a unique standard form by forcing the knot to first run through all overpasses, postponing any underpasses. While this algorithm may not terminate, it does provide an intuitive basis for an important class of knot invariants known as universal (quantum) knot invariants.

Universal invariants include the Jones polynomial and many of its generalizations and behave well under many topological operations on knots. We will illustrate these ideas with a computer implementation of a particular universal invariant that is distinguishes all knots in the Rolfsen table yet is computationally effective. Unlike all other known powerful knot invariants it runs in polynomial time in the complexity of the knot diagrams. This is joint work with Dror Bar-Natan and parts of our work appeared in arXiv:2007.09828 and arXiv:1708.04853.

Alexander Mednykh (Novosibirsk State University) 5/28 10am

Volumes of knots and links in spaces of constant curvature

We investigate the existence of hyperbolic, spherical or Euclidean structure on cone-manifolds whose underlying space is the three-dimensional sphere and singular set is a given knot or link. For two-bridge knots with not more than seven crossings we present trigonometrical identities involving the lengths of singular geodesics and cone angles of such cone-manifolds. Then these identities are used to produce exact integral formulae for the volume of the corresponding cone-manifold modeled in the hyperbolic, spherical and Euclidean geometries.

Thomas Fiedler (Universite P. Sabatier Toulouse) 5/28 8pm

1-cocycles and knot invariants

We introduce an appropriate moduli space for diagrams of classical knots and we lift the Chmutov-Khoury-Rossi-Brandenbursky Gauss diagram formulas for the coefficients of the Conway polynomial to combinatorial 1-cocycles on this moduli space. In contrast to the Conway polynomial they can distinguish mirror images of knots when they are evaluated on canonical loops in the moduli space, and at least as 1-cocycles they are sensitive to the orientation of the knots.

Sergei Chmutov (The Ohio State University, Mansfield) 5/31 8pm

Construction of links from Thompson's group

Vaughan Jones introduces an algorithmic construction of links from elements of Thompson's group and demonstrate the Alexander type theorem. His algorithm is highly inefficient. One of the main problems in this direction is to find an efficient algorithm. I will discuss a definition of the Thompson group and combinatorial description of its elements. Then I will explain the Jones algorithm and discuss the actual computational problems.

Sang Youl Lee (Pusan National University) 6/1 10am

Biquandle cocycle invariants for surface-links in $\mathbb{R}^4$ via marked graph diagrams

A surface-link is a closed 2-manifold smoothly embedded in 4-space $\mathbb{R}^4$. A marked graph diagram is a link diagram possibly with 4-valent vertices with markers. It is known that a surface-link can be described by a marked graph diagram modulo Yoshikawa moves. On the other hand, it is well known that the (bi)quandle cocycle invariants and shadow (bi)quandle cocycle invariants are defined for oriented classical links in 3-space and surface-links in 4-space by using broken surface diagrams and cohomology theory of (bi)quandles. In this talk, I would like to introduce representation of surface-links via marked graph diagrams and then discuss a method of computing (bi)quandle cocycle invariants from marked graph diagrams.

Hwa Jeong Lee (Dongguk University - Gyeongju) 6/1 8pm
Quantum knots and the number of knot mosaics

Lomonaco and Kauffman developed a knot mosaic system to introduce a precise and workable definition of a quantum knot system. This definition is intended to represent an actual physical quantum system. A knot (m,n)-mosaic is an $m \times n$ matrix of mosaic tiles (T_0 through T_{10} depicted in the introduction) representing a knot or a link by adjoining properly that is called suitably connected. $D^{(m,n)}$ is the total number of all knot (m,n)-mosaics. In this talk, we construct an algorithm producing the precise value of $D^{(m,n)}$ for $m,n \geq 2$ that uses recurrence relations of state matrices that turn out to be remarkably efficient to count knot mosaics.

Toshitake Kohno (Meiji University, The University of Tokyo) 6/2 10am
Quantum computation and homological representations of braid groups

In the theory of topological quantum computation unitary representations of the braid groups play an important role. In this talk I will explain a method to obtain unitary representations from homological representations of braid groups by investigating a relation to conformal field theory.

Hugh Morton (University of Liverpool) 6/2 8pm
Using knot invariants

Knot invariants come in many different forms. They may reflect geometric or combinatorial features of the knot. Calculating and comparing them is not always easy.

Strategies for constructing them and test cases for comparing them will be described. I will look particularly at the case of satellites and symmetric mutants.

The general guidance when looking at an invariant is to suggest and subsequently identify geometric features which it can show up. Equally, for more geometrically defined invariants the aim is to target their computation.