

Mathematical English

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1 Quotations about Writing

Warning: This author needs to read a copy of Strunk and White and take the advice of good writing to heart. – a referee (about myself)

The time to begin writing an article is when you have finished it to your satisfaction. By that time you begin to clearly and logically perceive what it is you really want to say. – Mark Twain

The wastebasket is a writer's best friend. – Isaac Bashevis Singer

Easy reading is damn hard writing. – Nathaniel Hawthorne

Style takes its final shape more from attitudes of mind than from principles of composition, for, as an elderly practitioner once remarked, "Writing is an act of faith, not a trick of grammar." ... Style is the writer. – Strunk and White

I think I can tell someone how to write, but I can't think who would want to listen – P.R. Halmos

Defend your style – P.R. Halmos

Word-smithing is a much greater percentage of what I am supposed to be doing in life than I would ever have thought – Donald Knuth

It strikes me that mathematical writing is similar to using a language. To be understood you have to follow some grammatical rules. However, in our case, nobody has taken the trouble of writing down the grammar; we get it as a baby does from parents, by imitation of others. Some mathematicians have a good ear; some not (and some prefer the slangy expressions such as "iff"). That's life. – Jean-Pierre Serre

When writing about transcendental issues, be transcendently clear. – René DesCartes

In order to translate a sentence from English into French two things are necessary. First, we must understand thoroughly the English sentence. Second, we must be familiar with the forms of expression peculiar to the French language. The situation is very similar when we attempt to express in mathematical symbols a condition proposed in words. First, we must understand thoroughly the condition. Second, we must be familiar with the forms of mathematical expression. – George Polya

<http://www.quotegarden.com/writing.html>

2 Rough structure of the lecture

1. General writing instructions (follow closely Strunk and White)

2. Mathematical writing instructions (including some hints on mathematical publishing)
3. L^AT_EX-nical writing instructions (if time permits)

This material is prepared during summer 2004 (Mathematical writing) for a minicourse at Osaka City University, Osaka, Japan and during spring 2010 (General writing) for a course at Keimyung University, Daegu, Korea.

As usual with lecture notes, they are incomplete, under construction, and contain typos and grammatical flaws (maybe they violate at least once half the rules they describe!) Corrections are always welcome!

3 General writing instructions

This exposition follows mainly Strunk and White, with some additions from Higham's book, and some of mine.

3.0 Names of grammatical elements

characters

.	dot, period, full stop	!	exclamation mark	} punctuation marks
,	comma	?	question mark	
:	colon	'	quote, apostroph	
;	semicolon	"	double quote	
...	ellipsis			

A stop character is one that can terminate a sentence – “.”, “?”, or “!”.

()	parentheses, opening and closing	[]	brackets, opening and closing	{ }	braces	} delimiters
< >	angle brackets		vertical bars			
-	hyphen	—	dash	—	long dash	} hyphenation
*	asterisk	+	plus	/	slash	} other characters (in math operators)
%	percent sign	—	minus	\	backslash	

words

prepositions: in, at, above, below, with, after, before, ...

conjunctions: not, because, while, if, and, or, unless, (un)like, ...

clauses

The dog <u>which I saw</u> was black.	not separated
This is true (<u>unless $x = 2$</u>).	separated by parentheses
The journal, <u>which appears weekly</u> , is popular.	separated by punctuation

3.1 Elementary rules of usage

3.1.1 Punctuation

Each sentence ends with a stop character, except if it ends on a quote, when the stop character may be placed before the quote, or on an ellipsis

3.1.2 A or An

- a – when next word starts with a consonant sound (not letter)
an – when next word starts with a vocal sound (not letter)

words beginning with a vocal letter but consonant sound: “European”, “university”, “user”, start with “yew”, so write “a ...”

Usually when word starts with a consonant letter, it is also pronounced. Exception is the unsounded “h” in “heir”, “honest”, “honor”, “hour” and their derivatives (“honesty”, “honorable”, ..., but in “heritage” the “h” is sounded!).

for abbreviations depends on their reading: e.g. “an LAPACK routine” (“ell-ay-pack”) but “a NAG routine” (“nag”).

3.1.3 Possessive singular of nouns

general rule: add ’s whatever final consonant:

Charles’s friend	box’s exterior
Burns’s poems	witch’s malice

exceptions:

- nouns ending on -ss get ’ only:
the dress’ model for righteousness’ sake
- also possessive of ancient proper names on -is or -es
Archimedes’ theorem
- Jesus’
- a few standard terms: for convenience’ sake

Pronominal possessives do not take ’ or ‘s’:

a friend of yours	a house of theirs
-------------------	-------------------

but indefinite pronouns do (both):

one’s rights	someone else’s umbrella
--------------	-------------------------

“its” vs. “it’s”: the first is a possessive, the second is a contraction of “it is”:

It’s freedom that we fought for.	(The <u>thing</u> we fought for was freedom.)
Its freedom was fought for.	(People fought for the freedom of <u>something</u> mentioned before.)

The “(s)” possessive in the form “A’s B” for “B of A” should be used only if “A” is short. “A” should not have the possessing noun followed by attributes: “the proof of the lemma on the last page” is not “the lemma on the last page’s proof”.

3.1.4 Enumerative comma

In a series of 3 or more terms with a single conjunction, use comma after each except last:

red, white, and blue
He opened the letter, read it, and made notes of its
content.

exceptions: it case the enumerated items are short, I find omitting the comma before second last item also OK.

red, white and blue

But less OK is:

He opened the letter, read it and made notes of its content.

In the following case, a comma is needed to resolve ambiguity of meaning:

A dictionary is used to check spelling, shades of meaning, and usage.

(If we omit the comma before “and”, “shades” will modify “usage”.)

Another exception occurs in names of companies (follow individual writing):

Little, Brown and Company (“& Co.”)

In the case of nested enumeration, one can replace the outer list commas by semicolons to avoid confusion.

The theorem is known to hold for symmetric or Hermitian matrices;
matrices of odd order not divisible by 3, 5 or 7; and for positive or
negative definite, and orthogonal matrices.

In general it seems, though, better to decompose such clumsy constructs into several shorter sentences, or typeset the outer enumeration as a displayed item list.

3.1.5 Parenthetical and non-parenthetical expressions

A *parenthetical expression* is one whose omission does not alter or spoil the main sense of a sentence. (This is not the same as a *parenthesized* expression, which we will treat later.) *Enclose parenthetical expressions in commas.*

Example:

The best way to solve this problem, unless it is too difficult, is by hand.

This rule is often hard to apply, because it is not easy to decide what expression is parenthetical. It depends on how much the phrase interrupts the flow of the sentence. So, the longer the phrase, the better the commas.

Example:

The answer you can imagine was negative.

The answer, as you might probably imagine, was negative.

I usually put comma for “too”, “though”, and “however”.

This, however, was unexpected.

I, too, attended the meeting.

(But: “I also attended the meeting.”; no comma.) Note also the other meaning of “however” as “in which way ever” or “in some (whatever) way”. The following two phrases say rather different things:

However, the problem can be solved by
more recent methods.

However the problem can be solved by
more recent methods.

Whether you decide to put commas, or not, never put one comma, but leave out the other:

Sandra’s husband, Prof. Nelson visited us yesterday.

My brother you will be pleased to hear, is in perfect health.

(both are bad).

Name or title in direct address is parenthetical, and needs commas:

If, Sir, you refuse, I cannot help you.

Well, Susan, now you have real trouble.

The abbreviations “etc.” (meaning “and so on”), “i.e.” (“that is”) and “e.g.” (“for example”) and abbreviations for academic degrees and titles after the name are parenthetical:

Letters, parcels, etc., should go here.

H. Fulsome, M.Sc., presided.

But: “Prof. A.B.”, “Dr. X.Y.” (no comma) Also, if “etc.” is preceded by a single object, a comma before “etc.” can be omitted (some writers still prefer putting it).

no comma for restrictive terms of identification:

Alexander the Great
The novelist Jane Austen
James Wright Jr. (Junior)

Dates:

February to July, 1982	(<u>but</u> I find “February to July 1982” also OK)
April 6, 1986	<u>but</u> : 6 April 1986
Wednesday, November 14, 1990	

restrictive and non-restrictive clauses

non-restrictive explains *one* thing more closely (this includes clauses with conjunctions for time and place) – put commas:

Example:

In 1952, when the university was founded, I was not born yet.
This book, which I bought recently, is not well-written.
Seoul, where I was born, became very crowded.

A sentence with a non-restrictive clause is equivalent to two separate sentences:

In 1952 I was not born yet. In that year the university was founded.
I was born in Seoul. It became very crowded.

restrictive explains *which* among several things is meant – no commas:

In the year when the university was founded I was not born yet.
The book which I bought recently is not well-written.
The city where I was born became very crowded.

Similar rule applies to participial phrases (starting with -ing form) or nominations:

My mother, living in the States, is a doctor.
People sitting in the rear could not hear well.
My brother Jim lives in the States.
My oldest brother, Jim, lives in the States.

exception is to omit commas for close relatives, even when they are unambiguous. (E.g., nowadays it seems strange to write comma after “my mother” or “my father” when they are followed by the respective names; in general, in case of doubt, better leave commas out.)

One or several subordinate clause(s) preceding the main clause require a comma.

Partly by hand, partly by computer, we could finally solve the problem.

3.1.6 Parenthesizing versus comma-separation

A *parenthesized clause* is a clause put into parentheses. A parenthesized clause should be parenthetical, but not necessarily the other way around. The difference between putting a clause into parentheses and setting it off by commas is that parentheses more strictly separate.

Parenthesized clauses should be grammatically redundant within their sentence, but also logically redundant within their context. That is, no logical ambiguity or inconsistency should be introduced into a sentence, or sequence of sentences, when parenthesized clauses are thought as deleted. In particular, *outside a parenthesized clause, no reference to its content should be made*, at least by explicit words whose correct use would require the clause to stand.

Thus the parentheses can be a tool of directing to what a reference in subsequent text is made.

Mike, as well as John, came only seldom. If he came, he was usually late.

Mike (as well as John) came only seldom. If he came, he was usually late.

In the first case, the reference “he” in the second sentence is ambiguous. Parenthesizing the clause involving John on the right makes it unambiguous, for one cannot refer from outside to what is inside the parentheses. So we now clearly say that Mike was usually late.

In the opposite direction, when a reference to a clause in parentheses is needed, the clause must be deparenthesized:

Mike (as well as John) came only seldom. If they came, they were usually late.

Mike, as well as John, came only seldom. If they came, they were usually late.

In this case, “they” clearly involves also John, and so he must get out of the parentheses to allow being referred to.

exception. One can refer into a parenthesized clause from another one, when

there is a clear logical parallel.

For a positive (respectively, negative) definite form, the norm of every non-zero vector is positive (respectively, negative).

In this situation, to keep the logical parallel visible, both parenthesized clauses should not be too far apart. Preferably, they should be in the same sentence.

Implicit references into parentheses might be acceptable. E.g., in a mathematical proof, an explanation in parentheses can be helpful, or needed somewhere later on to understand better a subsequent argument. There, it can be indirectly referred to by an expression like "as we have remarked/explained before". But in any event, locally or globally important information is not to be put into parentheses.

A *parenthetic clause in commas* should be only grammatically redundant, and only in a limited scope. This scope can be until the end of the governing clause, or until the following stop (".", "!", "?", ";") or semicolon. Which of both scopes is understood, is a disputable choice.

Mike, as well as John, came only
seldom, and if they came, they
were usually late.

Mike, as well as John, came only
seldom; if they came, they were
usually late.

In the first sentence it can be deemed legitimate or not to use "they" to refer to John as well. Certainly it is, in this case, at least clear who "they" are. The second alternative is definitely better. Consider the versions obtained by replacing "they" by "he":

Mike, as well as John, came only
seldom, and if he came, he was
usually late.

Mike, as well as John, came only
seldom; if he came, he was usually
late.

In this situation, the right version is more problematic, for a semicolon separates stronger, and hence terminates more visibly the scope of redundancy of the clause involving John. I would surely avoid this variant.

A parenthesized clause or expression should not start a sentence. In general it is better if at least some part of the main information of the governing clause comes before it, so that the reader is provided a direction. This is the more important, the longer the parenthesized clause is. Compare:

While (as I learned only acci-
dentally the next week) he had
left for a business trip, his fam-
ily stayed home.

While he had left for a business
trip (as I learned only acciden-
tally the next week), his family
stayed home.

This rule applies also to clauses in commas, but even more so to parenthesized ones, which are designated as more secondary.

The rule should be followed for whatever (in particular, long) parenthesized expressions.

A special case of this rule is when a noun has a long modifier. Thus long modifiers often better come after the noun, but not far apart from it (so that it is clear to what they refer).

Many (red, green, blue, sometimes also multi-colored) bands decorated the room.

Many bands (red, green, blue, sometimes also multi-colored) decorated the room.

The right version is better. Note that, were the clause non-parenthesized, on the right (after the noun) it would need commas, whereas on the left (before the noun) it would need none.

Many red, green, blue, sometimes also multi-colored bands decorated the room.

Many bands, red, green, blue, sometimes also multi-colored, decorated the room.

The placement,

Many bands decorated the room (red, green, blue, sometimes also multi-colored).

if not ridiculous, at least does not clearly convey what is meant.

Again, notice that for short parenthesized modifiers, the situation is rather reverse to above. Compare:

Many (red) bands decorated the room.

Many bands (red) decorated the room.

The right variant does not sound natural at all.

3.1.7 Place a comma before conjunction introducing an independent clause

Example:

These problems are difficult, and they can not always be solved.
These problems are difficult, but they can sometimes be solved.

Comma is used always for independent clauses starting with *as* (in the sense of “because”), *for*, *or*, *nor*, *while* (in the sense of “at the same time”).

Example:

Neither did he learn, nor did he write homework.
I was listening, while he was playing.

No comma is needed for a second conjunction is combination. (This occurs if the second clause starts with a subordinate clause.)

This problem is hard, but if we work hard, we can solve it.

Here there is no comma before the "if". Similarly do not separate by comma "or if", "but when", etc. When first conjunction is "and", there is an exception noted below.

If conjunction between independent clauses is "and", the comma before it can be omitted, if the subject of both clauses is the same, it is not repeated in the second clause, and if the relation between the statements is close. There should always be a comma before "but".

I tried to solve it, but failed.

I was experienced and could solve it.

but

I was experienced, and I could solve the problem. (repeated subject)

I came home, and then went shopping. (not directly related)

In the case we omit comma before "and", there should be a comma after it when followed by another conjunction.

We can work hard and, if time permits, solve the problem.

"We can work hard, and if time permits, solve the problem." is also OK (and essentially equivalent), but not "We can work hard, and, if ...". When comma is put after "and", and not before, then the subject can be repeated in the second main clause.

We can work hard and, if time permits, we can solve the problem.

This subject repetition in fact reads smoother than the previous example, and generally when the subordinate clause is long, making the subject drifting too far behind. But if the subordinate clause is left out, the previous subject repetition rule applies again, and we need the comma before "and".

We can work hard, and we can solve the problem.

3.1.8 Do not join independent clauses by a comma, but by a semicolon

If two or more clauses are grammatically complete (can stand as a sentence on their own), and not related by a conjunction, then use ";", not ",".

Mary Shelley's books are entertaining; they are full of interesting ideas.

It's nearly 7pm; we cannot return before dark.

But we need “,” when using a conjunction.

Mary Shelley’s books are entertaining, for they are full of interesting ideas.

It’s nearly 7pm, and we cannot return before dark.

As a general rule, “;” can stand where “.” also goes.

Mary Shelley’s books are entertaining. They are full of interesting ideas.

It’s nearly 7pm. We cannot return before dark.

The advantage of “;” over “,” is to avoid the conjunction, and over “.” to still suggest a close relationship (often cause-consequence) between the two statements.

Exception: when clauses are short and alike, or tone of sentence is easy and conversational, one can use a comma.

Here today, gone tomorrow.

I hardly recognized him, he was so changed.

Note that if the second clause is preceded by an adverb, such as “accordingly”, “besides”, “so”, “then”, “therefore”, or “thus”, and not by a conjunction, the semicolon is still required.

The student was absent too often; accordingly, his attendance score was not good.

I had never been in the place before; so I had difficulty in finding my way about.

But when the tone is less formal, and a close relationship between the clauses is to be expressed, I personally consider a comma instead of a semicolon sometimes also OK. This applies in particular in the case of “so” (when abbreviating “and so” or “and therefore”):

I had never been in the place before, so I had difficulty in finding my way about.

A comma for a semicolon can be useful for linking close consecutive steps of reasoning in a mathematical proof.

We obtain then $x = 2$, so $y = 3$, and then we see that $z = 5$.

(As in this example, in the case of three or more clauses, we avoid also repetition of “and”.) In general, however, this style appears somewhat loose. It is thus best, in writing, to avoid overusing “so” in such a manner. (For more on “so” see §3.4.6.) One possible alternative is to omit the word “so”, and begin the first clause with “as”:

As I had never been in the place before, I had difficulty in finding my way about.

Another example, common in scientific papers, where I consider a “;” unjustifiably replaced by a comma is something like “This is a well-known theorem, see [9].”

3.1.9 Do not break sentences in two

In other words, do not use periods where commas separate clauses, at least one of which is incomplete.

This was a hard problem. One that had not been solved for a long time.
One of the hardest problems of that time. It had not been solved for centuries.

In this case, replace the first period by a comma, and start the second clause with a small letter.

This was a hard problem, one that had not been solved for a long time.
One of the hardest problems of that time, it had not been solved for centuries.

Exception are emphatic words or expressions, which can serve the purpose of a sentence:

I wrote again and again. No reply.

3.1.10 Usage of colon

after an independent clause to introduce

- 1) a list of particulars
- 2) amplification
- 3) illustrative quotation

A ":" has more effect than a ",", and less separation than a ";".

Example type 1) A ":" should not separate a verb or preposition from its object:

We visited: two museums, the opera, and the city hall.	We visited the following places: two museums, the opera, and the city hall.
---	---

type 2)

This is not always true: look, for example, at $x = 2$.

type 3)

This reminds me of the proverb: "Easy come, easy go".

A colon is also used in formal constructs: letter openings, time, titles of books, Bible verses:

Dear Mr. Smith:
at 10:30am
Analysis: From Theory to Applications
Romans 10:8

3.1.11 Usage of dash

Remark: do not confuse a dash (–) with a hyphen (-); their grammatical function is rather different. The use of hyphen is discussed in §3.1.16 (line breaks) and §3.2.5 (combined words).

A dash can be used to set off an abrupt break or interruption of the flow of a sentence, a long appositive, or a summary.

Examples

- 1) His first thought after getting out of bed – if he had any thought at all – was to get back in again.
- 2) We heard some noise from the distance – a deep and rolling thunder.
- 3) The shorter days, the blowing wind, the falling leaves – all signs were present that winter was coming.

Use dash only when punctuation mark is not appropriate.

The concerns turned out justified – the answer was not correct – there was a mistake in the solution.

Typing errors – of the sort one sees in articles – occur often – this is why proofreading is necessary.

The concerns turned out justified: the answer was not correct; there was a mistake in the solution.

Typing errors, of the sort one sees in articles, occur often. This is why proofreading is necessary.

3.1.12 The number of a subject should match the number of its verb

‘Number’ here refers to the distinction between singular and plural.

– Number should not be affected by clauses put between subject and verb.

The proof, it all its detailed steps, are to be carefully checked.

The proof, it all its detailed steps, is to be carefully checked.

– When subject (or noun followed by a relative clause) is "one of" + plural form, then verb (in clause) must be plural.

He is one of the people who never come on time.

One of the most famous scientists who have tried to solve the problem is...

– singular verb after "each", "either", "everyone", "everybody", "neither", "nobody", "someone", or after "none", if it means "no one" or "not one":

Everybody thinks he is the best.

Although both methods are useful in general, neither works in this particular case.

None of us is perfect.

But if "none" refers to more than one thing or person, use plural:

None are as fallible as those who are sure they are right. (none = "no people")

– compound subject formed with "and" almost always requires plural:

He and his father are in good relation.

Exception are (often cliché-like) compounds which are considered a unit, like "bread and butter", "wear and tear", as well as compound subjects qualified by "each" or "every":

Fame and fortune was what he sought.

Every circle, triangle, and square is a geometric figure.

– a singular subject remains singular (and requires a singular verb) even when connected to other nouns by "with", "as well as", "in addition to", "except", "together with", "no less than", "along with".

The soup with the noodles was delicious.

The soup, as well as the noodles, was delicious.

His speech as well as his manner is objectionable.

– A linking verb agrees with the number of its subject:

What we need is a few more pairs of hands.

In this case, the subject of the underlined "is" is "what we need". The sense is as in:

The thing (which) we need is a few more pairs of hands.

(Note that the which-clause has no commas!) However,

A few more pairs of hands are what we need.

is better than the version with “is”. Here “a few...” is the subject. Using “is” seems still fine, when we keep the subject being “what we need”. Indeed, it is possible to have the subject after the verb (like in “Here is your book.”). But such (longer) sentences tend to be harder to read and interpret, and are useful only if strong emphasis is to be put on the first words (see §4.4.3). Thus in the last example, “are” is better.

– A gerund (-ing form of verb used as a noun) usually requires singular, a participle used as a modifier needs the number of the noun it relates to.

Reading books is welcome.

Reading students are welcome.

(In the first sentence, “reading” is a gerund meaning “the action of reading books”, in the second sentence, it is a modifier to “students”.)

– Some nouns that appear plural are usually considered singular and given a singular verb.

Politics is an art, not a science.

The UN Headquarters is located in N.Y.

But

The general’s quarters are across the river.

”Contents” can be singular or plural, depending on whether it is countable or not:

The contents of the jar is jam.

The contents of the jar are marbles.

– ”A few”, ”a couple of”, ”a group of”, ”a number of”, followed by plural nouns usually require a plural verb form, despite the singular indefinite article.

However, if ”couple” means ”man and woman in relationship”, the singular is used:

A couple of people attend regularly.

A couple of an American and a Korean attends regularly.

”Couple”, explicitly meaning exactly two, should not be used for arbitrary persons (instead use ”two people”, or ”two persons”), or for things, where ”pair” can be used. Compare:

Every point in the plane has a couple of coordinates.

Every point in the plane has a pair of coordinates.

Only the second statement is precise, for it says that that coordinates are exactly two.

Note that "couple" in the sense "more than one but not too many" is colloquial and to be avoided in formal writing.

"Group of" can be used also with singular verb, if the emphasis is to be put on the group as a whole, not on the individuals in the group.

A group of people attends regularly. (It is the same collective every time.)

A group of people attend regularly. (several people)

In mathematics "group" has a special meaning, and should be used as the object for itself, and not as a sloppy substitute of "several". Thus it requires a singular verb.

A group of matrices is not always commutative.

For "a number of" usually the emphasis is not on the number, thus plural verb should be used. Only in seldom situations is a singular verb needed:

A number of registrations gives information about voting turnout.

3.1.13 Use proper case of pronoun

– personal pronouns and "who" change form according to whether they act as object or subject

Example:

The culprit, we suspected, was he. (subject)

We suspected him to be the culprit. (object)

Give this work to whoever looks idle. (subject)

Give this work to whomever you see looking idle. (object)

In the first sentence of the last pair, "whoever" is the subject of the clause "whoever looks idle"; the object of "to" is this entire clause.

– When "who" introduces a clause, its form depends on its function in that clause.

Virgil Soames is the candidate who we think (He will win, we think.) will win.

Virgil Soames is the candidate whom we hope (We hope to elect him.) to win.

A pronoun in comparison is nominative, if it is subject of a stated or unstated verb. (In general avoid "understood" verbs by supplying them.)

Sandy writes better than I. (subject) Sandy writes better than I do.

But:

Polly loves cake more than me.
(object)

Polly loves cake more than she loves me.

Objective in the following phrases is right:

Whom should I ask?

A group of us taxpayers protested.

("Us" is an apposition to "taxpayers", object of proposition "of".)

- Use single (not reflexive) pronoun as subject.

Blake and myself stayed home.

Blake and I stayed home.

– the possessive case of pronouns comes usually as an adjectival modifier (my, your, ...), but also as a noun form (mine, yours, ...). They have to be kept apart according to their role in the sentence:

One of my answers and one of yours are wrong.

(In the first case, the possessive is an attribute to "answers", in the second case the object of the proposition "of".)

– gerunds (-ing form of verb used as a noun) usually require possessive case,
 participles (-ing used as a compound of form of verb) takes the objective case:

Do you mind <u>my asking</u> a question?	(my action)	$\left(\begin{array}{c} \text{Can I, in principle,} \\ \text{ask a question?} \end{array} \right)$
Do you mind <u>me asking</u> a question?	(that I ask)	$\left(\begin{array}{c} \text{Can I also ask a} \\ \text{question?} \end{array} \right)$

There is a subtle difference in meaning between the two forms, as indicated on the right. A similar example is:

Do you object to my driving on the icy roads?

Do you object to me driving on the icy roads?

The first phrase can mean driving in general. It could be used in a situation unrelated to actual driving. Or it can mean: "You don't like how I am right now driving on the icy roads?" The second phrase, in contrast, suggests that we are considering to drive, but have not departed yet.

3.1.14 Dangling participle

This is another one of the most common types of error.

A participle phrase in the beginning of the sentence must refer to its grammatical subject.

Example of this common error:

Adding π^2 on either side, the equation becomes $\pi^2/9 = x + y$.

In this example, the participle ("adding") does not refer to the subject ("the equation"). The sentence sounds like the equation is adding something, while it's us who are doing this.

Solution: make "we" (those who are adding) subject of main clause.

Adding π^2 on either side, we obtain the equation $\pi^2/9 = x + y$.

Similarly:

Being non-linear, it took a long time to solve the equation.

Being non-linear, the equation took a long time to solve.

Exceptions to this rule are some idiomatic participles:

Assuming $n \geq 3$, the function f_n is monotonous.

Considering the large dimension, the problem seems not worth an attempt.

Frankly speaking, this does not seem possible.

Roughly speaking, this means ...

Generally speaking, ...

Strictly speaking, ...

Participial phrases preceded by a conjunction or preposition, nouns in apposition, adjectives, and adjective phrases follow same rule if they begin sentence.

On arriving in Chicago, his friends met him at the station.

On arriving in Chicago, he was met by his friends at the station.

A great mind, they honored him with many prizes.

A great mind, he was honored with many prizes.

Young and inexperienced, the task seemed easy to me.

Young and inexperienced, I thought the task was easy.

Without attending class, the homework is difficult.

Without attending class, I found the homework difficult.

3.1.15 If-then clauses

The conjunction "then" requires a comma before it, but also that the preceding clause starts with "if" or "when". A common error in scientific writing is to start

the preceding clause with "let" or "assume" ("let-comma-then construction"). In this case "then" should start a sentence on its own:

Assume $n \geq 3$, then f_n is convex.	<u>If</u> we assume $n \geq 3$, then f_n is convex.
	<i>or</i> : Assume $n \geq 3$. <u>Then</u> f_n is convex.

Another logical error ("false if") is of the following sort:

If we want to prove that f is continuous, then Jones's theorem shows ...

In this case the problem is that Jones's theorem asserts something independent of our present intention. Two possible rewritings are:

If we want to prove that f is continuous, then we can apply Jones's theorem, which shows ...	To prove that f is continuous, we use Jones's theorem, which shows ...
--	--

A similar blunder is:

If we assume that x is real, a theorem of Jones states...

The formulation of the theorem is likely, and should be, independent of what we are doing. Presumably the theorem claims some conclusion under the assumption that x is real. It is better to write (depending on how we want to use the theorem):

If we assume that x is real, a theorem of Jones <u>implies</u> ...	A theorem of Jones states that <u>when</u> x is real, then...
--	---

Often "then" is omitted when starting a second clause, but this can be confusing. In the following example, the comma resolves a conflict of meaning:

If the iteration is quadratic convergence, problems can be solved easily.	If the iteration is quadratic, convergence problems can be solved easily.
---	---

Only the position of the comma decides whether "convergence" should be modified by "quadratic", or should modify itself "problems" – both options could, in principle, make sense. One should thus not overload the comma with a division task. Its misplacement would be far less confusing, had the second clause been started with "then". (See also rule 5 in §4.4.2.)

3.1.16 Divide words at line ends, according to formation and pronunciation

Divide when not enough space for whole word at line end.

- do not divide off one or, for longer words, two letters on any side.
- no general breaking rule; a few frequently applicable principles:
 - divide word according to formation:
know-ledge (not knowl-edge); Shake-speare (not Shakes-peare); de-scribe (not des-cribe); atmo-sphere (not atmos-phere)
 - divide "on the vowel":
propo-sition; ordi-nary; espe-cial; reli-gious; oppo-nents; regu-lar; classi-fi-ca-tion (three divisions possible); deco-rative; presi-dent
 - divide between double letters, unless coming at the end of the simple form of the word:
Apen-nines; Cincin-nati; refer-ring; but tell-ing
 - for consonants in combination best by examples:
for-tune; pic-ture; presump-tuous; illus-tration; sub-stan-tial (either division); indus-try; instruc-tion; sug-ges-tion
 - examine syllable-division in a few pages of a carefully printed book.

3.2 A Few Matters of Form

3.2.1 Colloquialisms

should be avoided in scientific writing. In general prosa they can be used, if their effect is well estimated, without extra attention drawn to them by quotes or italication.

3.2.2 Exclamation

In general, exclamation should be not be used for simple statements, where a period would naturally go:

It was a wonderful day!

It was a wonderful day.

Instead it better be used in phrases that require it:

What a wonderful day!

Hello!

In scientific writing, exclamation is to be used extremely sparsingly. A rare occasion where it might make sense, is to convey a surprising insight. Even in this case, a brief clause indicating the unexpectedness of the result can moderate the tone.

The computation for the whole polynomial took only 10% more than for its z^2 -term!

The computation for the whole polynomial took, very unexpectedly, only 10% more than for its z^2 -term.

Keep also in mind the potential confusion with a factorial when using "!" after mathspeak.

3.2.3 Heading

Say top 1/4 of first page should be kept for heading. Title should not be too long and should not end with a period. (It may end with "!" or "?".)

3.2.4 Opening and closing letters

It is important to learn how to start and finish a letter. These are things like "Hi Alexander" or "Dear Dr. Stoimenow". After witnessing repeated difficulties among non-native speakers, I include the following explanation.

Opening. First 3 rules of thumb:

- If you write to superiors, call them in one of the following ways:
 - proper name, preceded by either academic title ("Dr.", "Prof.", but not Ph.D., M.Sc., M.D., etc.), military rank, or "Mr./Mrs./Ms.",
 - "Mr./Ms." followed by administrative title,
 - "Sir/Madam" with no name or title,

and terminate the opening with a comma.

Dear Mr. President,

Mr. Chief of Staff,

General Smith,

but not

President Jones,

Chief of Staff Adams,

Mr. Prof.,

- There is never a comma between "Mr./Mrs./Ms." and name, academic title and name, military rank and name, or "Mr./Ms." and administrative title. (Note: There is a comma between name and "Ph.D.", "M.Sc.", "M.D.", etc., but these are not used in openings. "Ms." refers to a woman in general, while "Mrs." explicitly indicates, and intends some relation, to the fact that she is married. This is sometimes useful in private correspondence, but usually not in administrative matters, so it could be mostly avoided there.)
- There is never a comma or any other punctuation after "Dear", except if "Dear" is used alone (and this can be done only to intimate relatives).

There is some distinction in punctuation and its meaning between openings with predicates ("Dear", "valued", ...) and salutations ("Hi", "Hello", "Good morning"). The following describes a large part of the acceptable options.

- a) in the form salutation + name (in the following a 'name' is to refer to a noun which you call the recipient; it does not need to be a proper name),
 - After salutation, a comma is optional, and makes opening less direct. (So it is recommended after "Good morning", but less needed for "Hi" or "Hello".) There should be no other punctuation after a salutation.
 - After name, "!" is very informal,
 ", " is moderately formal (best),
 ":" is very formal (might be fine for "Good morning", but I wouldn't use it),
 "." in particular after a short opening construct can suggest authority over (and dissatisfaction with) recipient. It should be avoided, certainly to superiors, no punctuation is tolerable in informal talk, but otherwise less appropriate.
- b) in "Dear", "valued", etc., followed by name, or just a name,
 - "Dear", "valued", etc., are predicates of the following noun. Thus, in the same way as elsewhere, there is no punctuation between them and the name. Even when several predicates are used in succession before the name, it seems better not to set them off by commas.

Dear valued customer,

Dear distinguished guests,

- The rules for punctuation after name are as in a), only that now "!" is less usual, and ":" more appropriate (can be used in general by secretaries).

- The attribute "my" in an opening should be used very cautiously. It can make sense in close relationships: "My old friend," "My son,". But otherwise it can be perceived as an exponentiated version of the opening with a dot at the end. The simplest and most general rule is thus to avoid "my" in an opening consistently.
- c) when there is a salutation with no following name, the rules for punctuation after the salutation are like those after the *name* in a). They express a similar attitude:

Hi:	$\simeq$	Hi Steve:	(unusual)
Hello!	$\simeq$	Hello Steve!	(OK)
Good morning,	$\simeq$	Good morning, Mr. Smith,	(OK)

but "Dear," is much more intimate than "Dear (name)," and other attributes are uncommonly used without name.

- d) when there is a salutation followed by an attribute before the name, punctuation after the salutation is as in a), and punctuation after attribute and name is as in b). When opening phrase is longer, a dot at the end reads fine:

Good morning, dear honorable guests.

- e) An attribute and its noun are never separated by a salutation:

Dear, good morning, guest,

is always wrong. If attribute and name are followed by a salutation, the proper punctuation is to follow the name by a comma, and the salutation by ". ", ", ", or "!" (with their meaning as already explained).

Alexander, hello!	Dear guests, good morning,
-------------------	----------------------------

Closing should consist of a closing formula ("Sincerely yours", "Best regards", etc.), followed by an optional comma (recommended), and your name, followed by an optional period (not recommended). When closing formula and name are written without line break, a comma after the closing formula is needed.

Sincerely yours, Alexander	Regards
	Alexander

but not

Sincerely yours Alexander	Regards:
	Alexander

3.2.5 Hyphenation and joint words

In case some words belong together logically and are used commonly in this combination, they can be joined by a hyphen.

It is difficult to say when to put hyphen to build a compound word and when not. Here are some common examples.

hodge-podge self-confidence so-called

"Well" or "ill" + participle often looks (to me) better with a hyphen:

well-known theorem, ill-posed problem, well-written book, ill-judged statement.

Similarly do some compound participial attributes when the combination of the components is standardly used:

line-breaking rules, mind-blowing performance, peer-review(ing)
process, tree-shaped figure, computer-modelled design.

For the participial attributes following the noun, the hyphen is less compulsory, and both versions, with or without hyphen, seem in general OK:

The journal is peer reviewed.	The journal is peer-reviewed.
The figure is tree shaped.	The figure is tree-shaped.
The design is computer modelled.	The design is computer-modelled.
The problem is ill posed.	The problem is ill-posed.

But the noun form, when it makes sense, reads better without hyphen:

line break, peer review, tree shape, computer model.

Note also that in other cases, too, the hyphen might drop out, depending on the function of the words in the sentence:

This is a stand-alone objection.	(attribute to "objection")
I stand alone with my objection.	(verb + attribute to "I")
The second-order term is...	
This term is of second order.	

In the following combinations the hyphen disappeared, and they are written as one word:

schoolboy	wildlife	lifeguard
breakthrough	rightout	herewith
today	sunlight	

It seems that historically, two words are written first separately, then with a hyphen, and finally jointly. A dictionary is the most reliable reference what of these options is appropriate.

Note that some of these compounds still have the word-separated versions, which may convey a different meaning:

He was leading a wild life. I came here with my friend.

The pronouns obtained by combining "some", "every" and "no" with "body", "one", and "thing", are usually written jointly. Exception: "no one" – always separate. The following are instances where other combinations are written separately:

The police found <u>some body</u> it could not identify.	("some dead body")
Among a cube, a prism, and a sphere, <u>every body</u> looks different.	("3-dimensional object")
In the case of the x_i , <u>every one</u> is positive.	("each one")
<u>No body's</u> cells can fight this virus.	("no organism")

"Everyone" refers to a person, but "every one" (as an equivalent of "each one") to an object; "body" should be written separate, if it does not refer to a (living) person, or to his/her body.

Prefixes like "self-", "pre-", "post-", "pseudo-", "quasi-", "anti-", "non-", "bi-", "intro-", "retro-", "pro-", suffixes like "-like", "-wise", do not stand alone as a word (in their intended function), but are logically bound to the following/preceding word. They should be joined to it, directly or with a hyphen, regardless of the position in the sentence.

a pre-historical artifact,	a prehistorical artifact,
post-modernism,	postmodernism,
This domain is pseudo-convex.	This domain is pseudoconvex.

Which of either options is chosen depends on the dictionary; sometimes both forms seem acceptable. In "semi-indefinite" the hyphen helps separating the two 'i', but "posttraumatic", "preeminent", and "preemptive" seem to be widely used also (though not only) without a hyphen.

Prefixes like "self-" feel with the hyphen much more comfortable than others like "pre-" or "non-". In general, scientific writing tends to favor the non-hyphenated forms. But the hyphenated forms reveal the structure of the word better and might be easier to read, in particular to non-native speakers. Still both hyphenated and non-hyphenated forms will exist for all pre-/suffixes.

Many such terms are newly coined in mathematical papers, and do not have an established orthographical convention. Then the spelling is solely the author's choice. At least one should be consistent with one's writing for the same combination.

Some of these Latin modifiers can be used as independent words (also with rather different meaning), in which case they are written separately:

a post stamp
an intro to this track (“an introductory portion”)

Some modifiers may regain a meaning on their own also in rare idiomatic phrases, where they stand as separate words:

<u>post</u> factum	(Latin: “ <u>after</u> the act has been committed”)
quid <u>pro</u> quo	(Latin: “a favor <u>for</u> a favor”)
persona <u>non</u> grata	(Spanish: “a person who is <u>not</u> welcome”)

In enumerations of hyphenated constructs a suspensive hyphen should indicate binding of enumerated objects:

For integer-, real-, and complex-valued functions. . .
An A-, B-, or C-type domain. . .

There are some longer phrases which can be turned into nouns or attributes by hyphenating all words:

a do-it-yourself kit,
a state-of-the-art method,
a once-in-a-lifetime experience,
an attitude of we-can-do-as-we-please.

The last example suggests the disadvantage of such constructs. The longer, the more difficult they are to read and grasp (even although the hyphens give the logical binding). They also convey a less formal writing style.

3.2.6 Numerals

– do not spell out serial numbers, write in arabic or roman notation, as appropriate:

21st century	Chapter XII
August 9, 1988	Rule 3

Exception to this rule are numerals in quoted speech, in particular dialogues. It is fine to write:

”I arrived on August ninth.”
”In the year 1990, I turned twenty-one.”
”Read Chapter Twelve.”

Similarly to the previous example, references to theorems, lemmas, etc., in a paper should be written as numbers. Also, a reference to a value should be numeric:

The mean age is 43.

We see that x must be 2.

Exception is the number "1", for which "one" and "unity" can also be used:

z is of modulus one

a complex root of unity

(Beware, though, of the meaning of "one" as an indefinite personal pronoun.)

– small numbers should be spelled out when used as adjectives:

Two more problems occur.

the three lemmas

3.2.7 Plural forms

The possessive 's is sometimes used also in idiomatic plural forms:

do's and don'ts,

(but not "don't's")

pro's and con's

("pros and cons" also OK)

It is sometimes used also for plural of acronyms or mathspeak terms. My general attitude is that 's indicates possessive, and should be avoided as a plural construct even when no ambiguity arises. I thus prefer the version without apostroph or with a hyphen.

0s, 1s, *xs*, *ys*, LEDs;

0-s, 1-s, *x*-s, *y*-s, LED-s

In the following example, the plural-expressing 's can be avoided by quotes:

His speech contains too many
so's.

His speech contains too many
"so"s.

However, both versions seem tolerable.

Mathspeak plural forms should be used sparsingly, and can often be rewritten more smoothly (see rule 16 in §4.4.2).

3.2.8 Abbreviations

It seems that in scientific writing, the use of abbreviations like "i.e." ("that is"), "e.g." ("for example"), "w.r.t." (with respect to), and "w.l.o.g." (without loss of generality) is frowned upon. I often cannot refrain from using them, but explicitly declare the meaning of the last two somewhere in the beginning of a paper.

“I.e.” and “e.g.” should always be preceded, and usually be followed by a comma. They should not be written “eg.” or “ie.”, for each of them originates from two words (the Latin *exempli gratia* and *id est*).

“I.e.” should stand where “in other words” can also be used. In:

The fastest algorithm, i.e. Newton’s algorithm, converges quadratically.

the “i.e.” makes sense only if we mentioned previously that Newton’s algorithm (among several algorithms we discussed) is the fastest.

The abbreviation “cf.” should be written with one period; it comes from the Latin *confer*. This means “compare”, and so “cf.” should be used in that sense, and not in the sense of “see” (as in “cf. [6] for a discussion”).

The abbreviation “iff” can be spelled out as “if and only if”. In definitions it can be usually replaced by a simple “if”.

Non-standard abbreviations and acronyms are usually introduced parenthetically after the first occurrence of the terms they stand for.

Gaussian elimination (GE) is a Gaussian elimination (henceforth abbreviated GE) is...
method for solving linear equation systems.

Names. Initials in names should come either with a comma after the written-out family name, or without a comma before it.

H.S.M. Coxeter

Coxeter, H.S.M.

The comma version usually occurs only in references at the end of a paper, and should not be used in ordinary text.

– Initials should be followed by a dot. In certain cases one is allowed (or even demanded) to omit the dot. This, however, *to me* appears as a sloppiness, which I would allow only in informal writing. There should be no white space between initial and following dot.

– It is optional whether to leave white space between the initials with dot, and between last initial and name (when family name is written last). However, the same choice should be made between all initials, and if there is no white space between last initial and name, neither put one between other initials. Thus

H. S. M. Coxeter

H.S.M.Coxeter

is OK, but not

H. S.M. Coxeter

H. S. M.Coxeter

– When abbreviating compound names with a hyphen, both parts should be abbreviated with dot, and the hyphen retained without white space on either side. Thus “Jean-Pierre Serre” should be abbreviated as “J.-P. Serre”, not “J.P. Serre” or “J-P. Serre”.

3.2.9 Spacing

Do not put punctuation marks (period, comma, closing quotation marks) and closing delimiters (parentheses, brackets, braces) as the first character of a line. Similarly with opening delimiters as the last character of a line.

In general, a space can stand in ordinary text where a line break also goes. Accordingly, never put white space before, but always after punctuation marks and closing delimiters (unless they are followed by another character of these sorts). Exactly the opposite applies to opening delimiters.

A stop character can only be followed by white space, a closing quote, or a closing delimiter. The only exception to this rule are the dots in abbreviations, and multiple "!" and "?". (The more such occur, the more colloquial is the expression; usually such combinations have little use in scientific writing.)

In general, there should be no two punctuation marks in succession, even with white space in between. Exceptions are when one of the punctuation marks is a quote, the ellipsis "...", and the succession of two or more "!" and "?".

An ellipsis usually binds to the preceding word, and thus should have no space before, but space after it. This is the rule also if an ellipsis follows an opening quote or parenthesis.

I quote from the report: "...We He (...finally!) came.
investigated..."

3.2.10 Capitalization

- **The first word of every sentence is written capital.** There are hardly any exceptions tolerable to this fundamental orthography rule!
- Other words in a sentence are written small except for the pronoun "I", proper names, some words in titles, acronyms, and a few other words.
- articles, "a" and "the", are not capitalized
- Words derived from proper names usually inherit their capital letter:

Gaussian numbers, Hamiltonian system, Jacobian matrix, Hermitian form

Exceptions are "abelian" (coming from the Swedish mathematician Abel, but gradually accepted in uncapitalized form because of its frequent use) and the derivatives of "eigen-" (which comes from the German word "eigen" – "proper", but is not a person's proper name).

- When referring to a particular statement, a capital letter can indicate its proper name: the "Weak Convergence theorem" is something very different from a "weak

convergence theorem”. (However, even for the former the article ”the” is not part of the name, and thus write ”the Weak...”, not ”The Weak...”.)

– Similarly it seems widely recommended to capitalize the name of a statement when referring to a numbered statement (definition, conjecture, etc.) within some paper:

This is explained in Chapter IV. By Theorem 3, we see...

– Phrases in quotes can sometimes be capitalized and sometimes not. One meaningful (though not binding) rule is to capitalize if the phrase stands alone as a sentence. This usually happens only if the quote starts a sentence itself, or comes after a colon or a comma.

He said his friend ”will come soon”. He said: ”My friend will come soon.”

The same rule (bindingly) applies to capitalizing parenthetical phrases: if a parenthetical phrase is a *complete* sentence which is *not part* of an enclosing sentence, then it should be capitalized, otherwise not.

– In titles of sections, cited papers, books, etc., the first word is always capitalized. It seems common (and desirable?) to capitalize further words apart from proper names. Which words to capitalize seems generally a matter of taste. The common principle is apparently to capitalize those that convey the main meaning of the title; this applies to verbs and nouns, and most adjectives, though not to prepositions and most pronouns.

This this a book about the history of mathematics. This this the book ”The History of Mathematics”.

3.2.11 Parentheses

– A sentence containing an expression in parentheses is punctuated, outside of the marks of parentheses, exactly as if the expression in parentheses were absent.
– The expression within parentheses (part of an enclosing sentence) is punctuated as if it stood by itself, except that the final stop is omitted unless it is ”?” or ”!”.

The first word is not capitalized (unless a word which is always written capital, e.g., proper names, quoted speech, and titles).

This rule applies *regardless* of whether the parenthesized expression can stand

alone as a sentence or not.

I went to his house yesterday (my third attempt to see him), but he had left town.	I went to his house yesterday (it was my third attempt to see him), but he had left town.
--	---

He declares (and why should we doubt his good faith?) that he is now certain of success.

The book has a strange title ("What I Do Not Know" or something like that).	There are polite phrases ("Excuse me?") and impolite ones ("What do you want?").
---	--

– Accordingly, an expression in parentheses should not start with a punctuation mark, except an opening quote, or possibly an ellipsis. (Exception is the construct "(!)".) Usually, it should not end with a punctuation mark different from "?" or "!" or a closing quote (or an ellipsis). In some cases hyphens can be used to parenthesize prefixes and suffixes as in the following examples.

in (pre-)historical times	group(-like) elements
---------------------------	-----------------------

In rare cases, a comma may end a parenthesized expression in an enumeration.

The forest was full of green, (sometimes orange,) red, and yellow leaves.

However, commas should normally not stand before a closing parenthesis. In particular, if a parenthetical clause is parenthesized, commas are not needed on either side:

This man, whom I have never seen before, came yesterday.	This man (whom I have never seen before) came yesterday.
--	--

– When a parenthetical expression is a sentence on its own, the final stop comes before the closing parenthesis, and is not omitted even if a period. The sentence is started, as usual, with a capital letter.

I went to his house yesterday, but he had left town. (It was my third attempt to see him.)	He declares that he is now certain of success. (Why should we doubt his good faith?)
--	--

3.2.12 Quotations

– Not-too-long quoted phrases should be put into quotation marks. They can be introduced by a colon or comma:

The report of the committee says: "We examined the case, but found no clear evidence."	I remembered my friend's words, "There is always someone to help you."
--	--

In general a colon is more formal (and appropriate for documentary accounts) than a comma (appropriate for personal speech).

– Quoted phrases going over several lines, or verses or prose that need separation from the text, should be set as an own paragraph, with larger margins, sometimes in italics, and with no quotation marks.

– Single and double quotes can be used to indicate a quote within a quote. (The order of nesting seems to differ between UK and US.)

– A final punctuation mark should be placed outside the closing quote if it is not intrinsic part of the quoted text. (Here I disagree with Strunk and White!) Thus a period goes outside quotes, unless it ends a full sentence in the quote.

Phrases like "Hello!"	and	They said: "Your health depends
"What?" are colloquial.		on good food." But they did not
		say what is "good food".

Unlike "!" or "?", there is never a period before a closing quote, when the quoted phrase is not a complete sentence.

– When the quoted phrase ends with a period, and the surrounding sentence ends there, too, there is no need for a second period after the closing quote. If the surrounding sentence continues after the quote, there seems no uniform punctuation rule. Strunk and White suggest to replace the period with a comma, when a comma would follow in the surrounding sentence. In case no punctuation would follow, I obey the rule to still let the period stand (while other authors omit it).

"I have enough," he said.	Phrases like "I have enough." are
	impolite.

In the case of "!" or "?", I would punctuate the first sentence like this:

"I have enough!", he said.

(The succession of two punctuation marks different from quotes and "... " should be avoided; see §3.2.9.)

– Indirect speech introduced by "that" is not set in quotation marks and not treated as quoted text. The same is true if "that" is actually omitted.

He says that the situation looks not hopeful to him.

He says the situation is not hopeful.

The situation is not hopeful, he says.

However, note that some of these wordings go equally well with quotes, if this is what he actually said, and not only the meaning he tried to convey with his words:

He says that "the situation looks not hopeful".

"The situation is not hopeful," he says.

– In indirect speech certain contextual words must be reworded from direct speech. This holds for demonstrative and personal pronouns, and words like "here" (can be reworded as "at that place", "the place he were"), "today" ("on that day"), "tomorrow" ("the next/following day"), "yesterday" ("the day before").

He wrote: " <u>Here</u> <u>I</u> feel cold."	He wrote (that) <u>at the place he was</u> <u>he</u> feels cold.
My friend said: " <u>Let's</u> go <u>tomorrow!</u> "	My friend <u>suggested us to</u> go <u>the following day.</u>

3.2.13 Titles

Titles of books, journals, etc., are given in scientific texts usually as references at the end. The layout depends on the printing journal, book series, etc.

If a title is given inside text, usually it is italicized or put in quotes. In case a possessive (of the author's proper name) is used before the title, an initial "A" or "The" in the title should be omitted:

<i>A Tale of Two Cities</i>	Dickens's <i>Tale of Two Cities</i>
<i>The Age of Innocence</i>	Wharton's <i>Age of Innocence</i>

3.2.14 Ellipsis

– An ellipsis stands for part of a sentence which can or should be guessed by the reader and not be written explicitly, or which is not deemed important. Since the context of what is to be guessed must be given first, usually an ellipsis should not stand at the beginning of a sentence or a clause.

– In quoted speech an ellipsis can be used to skip a part of the speech (but without altering its sense!).

– If an ellipsis takes the role of "etc." in an enumeration, better replace it by the latter word (or "and so on", in particular in strict speech).

– In ordinary text use only the low ellipsis (...), not the centered one (⋯).

3.3 Elementary principles of composition

3.3.1 Choose a suitable design and hold to it

This means that, in most cases, you should have a clear structure in mind of what you are planning to write.

For example, a mathematical paper usually consists of a title, abstract, introduction (including statement of results), an optional preliminaries section, a main body (including proof of results), an optional open problem section, and a list of references.

3.3.2 Paragraph is a unit of composition

- Usually a paragraph conveys one idea, one topic. As such, it may be of any length.
- A paragraph of single sentence should be either a sentence of transition, or in a dialogue.
- Most commonly, a paragraph changes with speaker, so in dialogues, every new phrase appears in a new paragraph.
- It is helpful if a paragraph is started with a sentence that suggests the topic, or holds logically together the details that follow:

On the next pages, we describe some instances of this phenomenon.

Or it could be started with a sentence that helps the transition. It can be related to preceding content using words like "again", "therefore", "for the same reason".

- A very long paragraph could be broken into two without logical sense, just to make a pause of speech. This is in particular important in technical writing of complicated arguments in proofs. But, on the other hand, the succession of many short paragraphs can create the impression of display advertising, and distract the reader's attention.

3.3.3 Use active voice

Prefer active ("X did Y") to passive voice ("Y was done by X"). Active adds life and movement to writing, passive in excess loosens the connection to the reader by suggesting impartiality of the writer to the subject.

I shall always remember my first visit to Boston.	My first visit to Boston will always be remembered by me.
---	---

The passive phrase is longer, and less direct. Similarly,

A numerical example is given to illustrate the above result.	We give a numerical example to illustrate the above result. <i>or:</i> The following numerical example illustrates the above result.
Minimality of y was verified by checking...	We verified minimality of y by checking...
<i>or:</i> Verification of the minimality of y was achieved by checking...	

As the last example shows, often phrases such as "was" + one of "performed", "achieved", "carried out", "constructed", "accomplished" can be rewritten in active.

Other examples where an active replaces perfunctory phrases like "there is" or "could be heard":

At dawn the crowing of a cock could be heard.	The cock's crow came with dawn.
There were a great number of dead leaves lying on the ground.	Many dead leaves covered the ground.
The reason for the difficulty of these problems lies in their high dimension.	Their high dimension makes these problems difficult.

However, in certain cases the passive is useful, depending on what emphasis is to be placed in the statement.

Van Gogh's work was not properly acknowledged during his lifetime.	The painters of his age did not properly acknowledge Van Gogh's work during his lifetime.
--	---

Now both phrases make sense, depending on whether we put Van Gogh's work into focus, or the painters of Van Gogh's age. Examples where the passive allows the desired emphasis are:

The first proof of this result was found by ...
The subject has been given great attention in recent research.

In certain cases it is explicitly helpful to use passive to blur the origin of an action:

Several inexact measurements have been discovered during our experiment.

(in opposition to: "We measured inexactly several times during our experiment.")

- 3.3.4 Put statements in positive form (SW 15 u.c)
- 3.3.5 Use definite, specific, concrete language (SW 16 u.c)
- 3.3.6 Omit needless words (SW 17 u.c)
- 3.3.7 Simplification (H 4.28 u.c)
- 3.3.8 Avoid a succession of loose sentences (SW 18 u.c)
- 3.3.9 Express co-ordinate ideas in similar form (SW 19 u.c)
- 3.3.10 Keep related words together (SW 20 u.c)
- 3.3.11 In summaries, keep to one tense (SW 21 u.c)
- 3.3.12 tense (H 4.30 u.c)
- 3.3.13 Place the emphatic words of a sentence at the end(SW 22 u.c)

Warning: In scientific writing, the opposite policy is often more helpful. See rule 2 in §4.4.3.

... [TO BE COMPLETED]

3.4 Words and Expressions Commonly Misused and Misspelled

3.4.1 Words Commonly Misspelled

The following are lists, adapted from Strunk/White and Higham, of misspelling-prone words and their common misspellings:

	Philip	accidentally	advice/advise	affect/effect
beginning	believe	benefit	challenge	criticize
deceive	definite	describe	despise	develop
disappoint	duel	ecstasy	existence	formerly
humorous	hypocrisy	immediately	incidentally	latter
led/lead	lose/loose	marriage	necessary	occurred
parallel	playwright	preceding	prejudice	principle/principal
privilege	pursue	repetition	rhyme	rhythm
ridiculous	seize	separate	shepherd	siege
similar	than/then	too/to/two	tragedy	tries

Correct/Intended	Misspelling
analogous	analagous
criterion	criteria*
dependent	dependant*
discrete	discreet*
explicit	explicite
Frobenius	Frobenious
in practice	in practise*
implicit	implicite
led (past tense of lead)	lead*
lose	loose* (very common)
phenomenon	phenomena*
principle	principal*
preceding	preceeding
proceeding(s)	proceeding(s)
propagation	propogation
referring	refering
separate	seperate
succede	succeed
supersede	supercede
zeros	zeroes

The asterisked words are correct words on their own.

- "Criteria" and "phenomena" are the plural forms, and
- "lead" the present tense of "led".
- "Discreet" means "judicious, prudent",
- "advice" is the noun of "advise",
- "principal" is a head master at school,
- a "dependant" is a person (wife, child) you care for at your home,
- "dependent on" but "independent of"
- "loose" means as an adjective "not tied/tight", "not bound or fixed", "relaxed", "sloppy", and as a verb "to untie"
- "practise" is a (UK style) verb corresponding to the noun "practice" (in America, "practice" is used also as verb)

Here is a particularly confusing example of misspelling (all phrases are correct for themselves):

much to lose	("many things that could disappear", what is usually meant)
much to loose	("many things to untie")
much too loose	("completely untied or sloppy")

These examples are blunders based mainly on mere misspelling. The next subsection contains further, and more subtle, instances of misused words, where the meaning and/or grammatic function is misunderstood.

A few further words are given in §3.5. They are normally thought to be used well by native speakers, and errors in their usage are assumed to result primarily from unfamiliarity with English.

3.4.2 Words and Expressions Commonly Misused A-B

-able. A suffix that can be added to verbs to produce an adjective of a meaning like "that can be ...ed". Not all are bad, but all are suspect. "Seeable" means "visible" or "obvious", and "reasonable" means something different from "can be reasoned (concluded)".

aggravate and irritate. The first means "to make worse", the second to "annoy", "disturb", "confuse". A person is irritated, but a situation aggravated.

adjectives ending on "-al" and nouns ending on "-age" tend to have a more abstract meaning than their counterparts without this ending. The following examples show the difference:

The <u>usage</u> of the word is difficult to describe.	(in general)
Here the <u>use</u> of Euler's method is not recommended.	(in this particular case)
This <u>algebraic</u> calculation shows...	(concrete)
<u>Algebraical</u> methods are more efficient for this type of problem.	(in general)

allude means to suggest implicitly (unlike "refer", which is explicit). Not to confuse with "elude", meaning "escape", "slip out of somebody's hands".

allusion. The noun of allude, an indirect mention. Not to confuse with "illusion", which is "an unreal image", "a false impression".

alternate and alternative. The former means "changing constantly from one to another", the latter means "one of several options".

While he was writing his thesis,	This method could not be used to
the student alternated between	prove the theorem, so an alterna-
excitement and frustration.	tive was sought.

among and between. For two things or people, always "between". For more, usually "among", but if the individual position of each of them is considered, then use "between".

The prize money is divided	The agreement is between the six
among the team players.	heirs.

Another case of using "between" for more than two things is for location (in the mathematical sense of 'in the convex hull of').

Zurich lies somewhere between Vienna, Paris, and Rome.

and/or. A colloquial shortcut that often damages the sentence and leads to confusion and ambiguity. It should be strictly avoided in rigorous mathematical reasoning. An example of rewriting is:

Will the algorithm be efficient for	Will the algorithm be efficient
ordinary and/or partial differen-	for ordinary or partial differential
tial equations?	equations, or for both?

anticipate. If meaning "expect", then replace by the latter. In finance, has a different nuance:

My brother anticipated the upturn of the market.

Did he just expect this, or with this expectation in mind, also took action in advance, like buying stock?

any In questions (expecting a negative answer, in opposition "some") generally fine. But in statements can mean both "some" and "each/every". Thus, in particular in mathematical writing, it is better replaced by one of the latter words.

as to whether. "whether" is enough

as yet. In the sense of "up to now", "as of now", can be replaced by "yet", except in the beginning of the sentence. There "Yet" has a different meaning ("despite everything").

Yet he has not succeeded.	(Despite everything ...)
As yet he has not succeeded.	(So far ...)

being. Unnecessary in "regard ... as being"

both. In "both A and B", A and B should be grammatically equivalent. That is, a grammatical element common to both A and B, and starting both of them, should either be put into both, or moved out before the both/and-phrase. Sometimes both options are fine, and one has to balance between weakening the grammatical

binding by moving out and the repetition when keeping in. Consider the case of a verb (binding weaker) and a preposition (binding stronger).

both in the left and the right quadrant	both in the left and in the right quadrant in both the left and the right quadrant
He both plays soccer and tennis.	He plays both soccer and tennis. He both plays soccer and plays tennis.

In the first example, on the left A has a preposition but B not. The two options to rewrite, on the right, are to either put the preposition into both A and B, or take it out in front of the both/and-phrase.

Note that when a preposition comes before a both/and-phrase, the speech often sounds rather formal. This is good for mathematical writing and e.g. for law, but should be avoided when a more relaxed style is strived for. See also nor.

The second example is with a verb. In this case the rewriting repeating the verb (unlike a preposition) sounds less elegant.

An article binds too strongly to be moved out of a both/and-phrase (like "in the both left and right quadrant").

but. Unnecessary after "doubt" and "help". In

I have no doubt but that...	I have no doubt that...
-----------------------------	-------------------------

the first sentence means actually that "... " is the only thing I doubt (everything else is fine). In

He could not help but see that...	He could not help seeing that...
-----------------------------------	----------------------------------

both sentences mean "He could not avoid/resist seeing that...", but the variant without "but" is slightly better.

Avoid succession of many "but"s, for they alter direction of the speech. This can be smoothed by replacing a "but"-clause by a sentence that starts with "still" or "nevertheless":

This algorithm was widely used, but we expected it to be very inef- ficient. But we could do the com- putation.	This algorithm was widely used, but we expected it to be very in- efficient. Nevertheless, we could do the computation.
--	--

Even better repeated change of direction is amended by changing the order of

clauses. Two variants to rewrite the above example:

This algorithm was not expected to be efficient, but it was widely used. Indeed, we could do the computation.	This algorithm was not expected to be efficient. Still, it was widely used, and we could do the computation.
---	--

by means of and in terms of. Sometimes useful in mathematics, e.g. as "expressing A in terms of B". But in prosa act as pieces of padding, and often redundant.

The job was unattractive in terms of salary.	The salary made the job unattractive.
This is achieved by means of a technical procedure.	This is achieved by/using a technical procedure. or: A technical procedure achieves this.

3.4.3 C-E

character and nature. As vague terms of specifying a thing, often redundant.

A problem of subtle character/nature.	A subtle problem.
---------------------------------------	-------------------

If "nature" is to mean "natural landscape, flora, fauna" in expressions like "a lover of nature", "poems about nature", then make precise, unless following explanation does so.

compare with and compare to. "Compare to" is often emotionally motivated and means "view similar to" or "put (almost) equal with". "Compare with" analyzes similarities and differences between two things, and is what is usually done, and needed, in scientific writing. Thus comparing values of variables, etc., use "with", not "to".

His achievements were so great, that he was compared to Napoleon.	We now compare this algorithm with the previous method.
---	---

Usually the expression "compare A and B" is meant as "compare A with B".

compose, comprise, constitute. "Compose X" means a human to make up (usually with some creativity) X, out of some parts Y. Then "Y constitutes X" or "X

comprises Y". For example:

The lecturer composed the course out of three topics.	(He put three topics together to create the course.)
The course is composed of three topics.	(<u>not</u> : is comprised of)
The course comprises three topics.	(The course includes three topics.)
Three topics constitute the course.	(Three topics make up the course.)

contain, include and involve. Very often the separation between "contain" and "include" is by whether the relation is logical (include) or physical (contain).

The jar contains sweets.	The price includes the delivery.
--------------------------	----------------------------------

In "The documents include. . .", the content is regarded as the logical information it provides, thus "include" is used. In "The envelope contains documents," the documents are regarded as physical content.

Sometimes in mathematics writing, "include" and "contain" are separated by whether it is an element or subset relation (see rule 11 in §4.4.3).

"A involves B" means that A is some process of human activity (usually evolving and changing with the time) and that B is, in most cases, either part of this activity or a person responsible for it:

The duty involves project work.	The project involves many specialists.
---------------------------------	--

Thus:

I am included in the group of speakers at the conference.	I am involved in the conference as a speaker.
---	---

In the left sentence, logic suggests that "contained" is also possible. But avoid using "contain" when the object is a person, for it tends to treat the person as a physical thing and sounds impolite. Thus, instead of saying "The car contains five people," say "Five people sit in the car".

In the case of a tool or material that played a role during an activity, "involved" can be occasionally also ok; but the dividing line is fine, and one needs some feeling. A car can be involved in an accident and documents in an investigation, but a blackboard not in a conference talk session.

The following summarizing example should help keeping the difference in mind.

The driver is contained in the delivery.	(He is in the boxes of delivered goods.)
The driver is included in the delivery.	(We send him along with the goods at your disposition.)
The driver is involved in the delivery.	(He does part of the work needed for the delivery.)

due to and owing to. "Due to" means "caused by" or "attributable to", while "owing to" should be used where "because of" also goes.

Owing to a repelling fixpoint, there is no convergence.	The lack of convergence is due to the existence of a repelling fixpoint.
---	--

"effect" as a noun means result or consequence of a change, while "affect" is a verb and means to influence, or bring about change in.

The multiple roots affect the convergence of the iteration.	The effect of multiple roots is to slow down the convergence of the iteration.
---	--

"Effect" can also be used as a verb in the sense "cause an effect" (e.g. "effect a change" meaning "have as a result a change"), but this verb usage is rare, hardly needed, and not recommended in scientific papers.

effective and efficient. Similar in meaning but not equivalent. "Efficient" emphasizes the performance during an action, "effective" its final result. Thus an efficient algorithm is one whose performance criteria (speed, memory usage) are good, while an effective algorithm is one which gives a (desired) output. Similarly, an efficient speech means that one does not waste time with unnecessary things while speaking, an effective speech means that people understand (easily and well) what one says.

A similar pair is deffective and defficient. The former means "bringing about bad result or influence", the latter "being insufficient, or flawed (inside) in some way". Thus one's knowledge, or a scientific theory or method, can be defficient, but cheating, corruption, etc., are deffective practices.

else and otherwise. Often regarded as equivalent. In general, "else" is preferred for its brevity. However, whenever both words are possible, "else" is more colloquial, and there are some differences in usage.

After words such as 'anywhere', 'someone', 'what', 'everyone', 'everything', and 'everywhere' naturally comes "else".

We had nothing else to do on those long trips.	As I try to be truthful, I expect everyone else to be truthful.
--	---

"Otherwise" should stand for "if this is not the case, then ...". Compare:

Who else would like to go?

Who otherwise would like to go?

In the first case, further people wanting to go are sought. In the second case, the meaning is: "The situation is so hard, that you are the only one who could go. If you do not want to go, will anybody at all?"

An "else" which is not in "or else", and is used as a conjunction starting a clause, almost always demands preceding "if/when"-clause, and "then"-clause: "If A, then B, else C." If "B" is long, replace "else" by "otherwise" and precede by a semicolon.

If $x = 2$, the equation is easy to solve using this method, else there are problems.

If $x = 2$, the equation is easy to solve using this method; otherwise, there are problems.

A construction of the type "In the case of A, we have B, else C." seems also fine, but still colloquial, and even this sentence reads better when "else" is replaced by "otherwise".

An "Else" or "Or else" clause should not start a sentence. In that case, they are to be replaced by "otherwise".

Let us consider the case $x = 2$. Or else the equation is hard to solve.

Let us consider the case $x = 2$. Otherwise, the equation is hard to solve.

equal with and equal to. "With" is used in the qualitative sense of "not better or worse than" (for property, evaluation, achievement), and "to" in the quantitative sense of "not more or less/fewer than" (for value, amount, number). This way the choice of preposition is often, though not always, *opposite* to "compare".

Comparing Mr. X to Mr. Y, we think that Mr. X (achievement)
is equal with Mr. Y.

Comparing x with y , we find that x is equal to y . (value)

Comparing the new algorithm A with the old one (evaluation)
B, we find that A is approximately equal with B
in performance.

etc. Used, preferably surrounded by commas, at the end of an enumeration or quotation, and means "and the rest" or "and so forth". It should not be used if one of these would be insufficient, that is, when the preceding enumeration leaves out important features the reader would be left in doubt about, or the rest of the quotation conveys some important point.

3.4.4 F-I

facility. A formalistic word, often replaceable by the concrete type of building it refers to: prison, hospital, school. Makes sense in plural, e.g. in "recreational facilities".

fact. In "the fact that ..." often redundant. "Fact" should be used only for directly verifiable matters, e.g. that an event occurred on a given date, or a theorem holds. Should not be used for conclusions depending on personal judgement or public opinions, no matter how widely accepted they are. That Napoleon was the greatest general of modern history, or the climate in California is pleasant are no facts.

factor. As an indicator of cause, often redundant and used for wordiness. Possible confusion with mathematical meaning.

The main factor for their success	They succeeded mainly thanks
was. . .	to. . .

farther and further. "Farther" refers to distance, "further" to time (in the continuation of a process) or quantity. One jumps *farther*, but investigates a problem *further*.

fewer and less. "Fewer" refers to number (countable), "less" to size, amount or quantity (not countable), or mathematical value.

The zeros of f are less than those of g .	(When $f(x) = g(y) = 0$, then $x < y$.)
The zeros of f are fewer than those of g .	$(\#\{x : f(x) = 0\} < \#\{y : g(y) = 0\}.)$

As a general rule, use "less" with singular nouns, "fewer" with plural nouns: less computation, fewer results. "Less troubles" means "less serious troubles", while "fewer troubles" means "a smaller number of troubles".

fix. The meaning as "repair", "mend" is well-established (in US), but slightly colloquial. It should not be overused in scientific writing. Beware of confusion with the other, original, meaning, which is more common in mathematics, and is "to make firm", "to place definitely".

feature. Similar status to "factor": redundant, and avoidable by rewriting.

firstly, secondly, thirdly. Use without -ly.

folk. Stands for "a people" most properly in a cultural or ethnographical context: "folk craft", "folk art" (but "folklore", written jointly). Otherwise, in the sense of "those who inhabit a place, country, etc.," to be replaced by "people". The plural "folks" is too colloquial, and should be avoided in any sort of formal writing. See also the entry for *people* below.

get. A verb favored for brevity, but often inelegant. Avoid (at least continuous) use in mathematical writing. In the sense of "acquire" can be replaced by "obtain", in the sense of "turn into", can be replaced by "become". In the sense of "understand" (like in "do you get it?") colloquial and not to use in formal writing.

In the form "has got" (in the sense of "possesses"), often "got" can be omitted. The past form "gotten" should be avoided. In the sense of "obtained", it should be replaced by the latter.

hardly. Built from "hard" by the adverbial suffix -ly, but changed in meaning. Now the adverb of "hard" (in the sense of "difficult", "painstaking") is "hard" itself, "hardly" means "almost not" or "unlikely".

Thus "I work hard" is the equivalent of "I do hard work", whereas "I work hardly" means "I do almost no work". And "He will hardly come" means "He will unlikely come" or "I don't expect him to come", and not that his arrival is a painstaking adventure.

however. In the sense of "though", "nevertheless", enclose in commas and avoid starting too often sentences with it. If not enclosed in commas, means "in whatever way" (see §3.1.5).

imply and infer. "A implies B" in mathematics means that A has B as a logical consequence. If the subject is a person, "imply" means to "suggest or indicate indirectly". (This is similar to "allude", though one alludes to an object but implies a statement or opinion.) "Infer" is only done by human beings and means to deduce or conclude from evidence at hand.

I implied that there might have been some error in the experi- ment.	We inferred from the data that there was some error in the ex- periment.
--	--

important and interesting. As qualifiers to use sparsingly in scientific writing. Say why something is important, interesting, etc., instead of calling it so. Sometimes simply redundant, as in phrases like "it is interesting to recall/remark that..."

inside and within. Similarly to "contain" and "include", the best way to keep them apart is by whether the relation is physical (inside) or logical or temporal (within). In the expression "Inside the field of mathematics, ...", the field is meant as the (physical) community of people doing mathematics.

Often, in particular when the separation between physical and logical is not clear, a simple "in" will also do. Note, however, that in a temporal context, "in" refers more strongly than "within" to the end (rather than a point during) a given time period from the time of speaking.

It's 9pm, thus midnight is in 3 hours.	He will come until midnight, i.e., within 3 hours.
---	---

irregardless should be "regardless". The error results from taking the negative twice, first by the suffix -less, and then by the prefix ir-

-ize. A suffix put on nouns to create a verb. It means "put into" or "make into" that noun. Many useful words, like "summarize", are built this way, but beware of coining new ones in excess, like "containerize". Often an appropriate verb already exists, e.g. "use" instead of "utilize".

3.4.5 K-O

kind of and sort of. In the sense of "rather" or "something like" colloquial and to avoid in formal writing. Restrict use to their literal sense: "I dislike that kind of publicity", "Tuna is a kind of fish."

lay (transitive, requires an object) should not be mixed up with lie (intransitive). I *lie* down to sleep, but a hen *lays* eggs. The principal parts (most common verb forms) of "lie" are:

lie (present), lay (past) and lain (past participle); lying (present participle).

The principal parts of "lay" are:

lay (present), laid (past) and laid (past participle); laying (present participle).

The main problem is that the past of "lie" is the same as the present of "lay".

In the second meaning of "lie" as "not to tell the truth", the verb is regular (lie, lied, lied), but the present participle is the same (lying) as for the first meaning.

leave and let. "Leave" expresses more than "let" abandoning an object by taking physical or logical distance. I can *leave* a topic of research for a while (turn my attention to something else), or my home in disorder (when getting out), but I *let* an item in the state it is (because it is not necessary for this to get away from it).

When followed by verbs, "leave" needs either the to-infinitive or the -ing form. Note the difference between the two: the former indicates that the left object/person had not started yet the indicated action at the time he/she/it was left; the latter indicates that he/she/it had.

I left him standing in the mess he created.	I left him to clean up the mess he created.
---	---

"Let" should be best used with the simple infinitive. A usage with the to-infinitive in a sense similar to "leave" looks also possible, but I see very few such situations

where "leave" would be less appropriate.

I let him go.	(I stayed, and allowed that he go.)
I left him going.	(When he left heading for one place, I went off to another.)
I left him to go.	(I left him with the task/plan to go, but at the time I went away, he was still staying.)
I let him to go.	(We were still both staying, but he had planned to go later, and I gave up persuading him not to do so.)

like should bind to nouns and pronouns, not serve as introduction of whole clauses and phrases. In the latter case, use "as".

This was, like everything so far, done well.	It was an exciting feeling, as we had often experienced in the old days.
He works like a robot.	He works as a robot does. He works as if he were a robot.

line. In the sense of "course of procedure, thought", in particular in the phrase "along these lines", or "along the same lines", overworked.

meaningful. Similarly to "interesting", etc., a vague, and often unnecessary, qualifier. Say better what is meaningful.

most. In the sense of "the majority of", "the greater part of", do not replace for "almost". "Most" requires "of" and some uncountable noun, or a countable noun in plural. It expresses a lower degree of completeness than "almost". "Almost" is often followed by "all", "everybody", "completely", or a verb.

most of the time	almost all the time
Most shops close on Sundays.	Almost all shops close on Sundays.
most people	almost everybody
I finished most of the work.	I almost finished the work.

Note the difference of usage of "most" with "people" in its meanings as a singular (uncountable) and as a plural (countable) noun.

Most of the people of the country is nomadic.	Most people in the country are nomads.
---	--

nice. Another, in scientific writing often useless, qualifier.

nor. Requires "neither". Do not replace for "or" after a negative expression.

He cannot eat nor sleep.

He cannot eat or sleep.

He can neither eat nor sleep.

Neither can he eat nor can he sleep.

Similarly to "both ... and...", there is some grammatical equivalence required between A and B in "neither A nor B".

He can neither play soccer nor volleyball.

He can play neither soccer nor volleyball.

He was neither in the house nor the garden.

He was neither in the house nor in the garden.

The first example on the left (moving a verb into A but not B) sounds ok (more ok than if "neither/nor" is replaced by "both/and"), but in either case the right sides sound at least equally fine, if not better.

noun used as verb. Not all are bad, but all are suspect. To chair a meeting and debut a theater performance is probably ok, but less so to gift (instead of to give) a present, to loan (instead of lend) a book, and quite certainly not to headquarter (instead of have one's headquarters) in New York.

one/everyone. "One" in the sense of "a person" expresses stronger than "everyone" that the speaker belongs to the indicated group. Thus "one" should not be referred to (at least in the same sentence) by pronouns in third person ("he/she" or "his/her"). Use "one" instead, or rewrite the sentence if too many "one"s occur.

One must watch one's step.

(not his or her step)

In contrast, "everyone" can be used with a pronoun, and should not be mixed with "one". First, second or third person are all possible depending on the context.

Everyone must watch his or her step.

Everyone watch your step!

Everyone must make sure we can start on time.

"One" conveys also a greater sense of generality. Thus avoid "one" when talking about a specific situation.

On slippery trails, one must watch one's step.

When we walk this trail now, everyone should watch his step.

opportunity and possibility. The former should be used when (and only when)

one (always a person) can do something good/positive for oneself or others; otherwise, use the latter.

There is an opportunity to discuss with other scientists.

There is a small possibility that I am late.

There is a possibility that it rains tomorrow.

There is a possibility that the sequence (x_n) does not converge.

-oriented. Creates heavy and formalistic compound words and can often be rewritten.

An application-oriented method. A well-applicable method.

A reader-oriented style. A style appealing to readers.

3.4.6 P-S

partially and partly. The former refers to a condition or state in the sense of "to a certain degree", the second carries the idea of a part as distinct from the whole – usually a physical object.

The sun is partially eclipsed.

The sun consists partly of hydrogen, partly of helium.

Mr X partially contributed to the proof.

My duty involves teaching only partly, otherwise I do research.

people. It has many meanings. One is as a political term "the people". As such it must be distinguished from "the public". The people elects a president, the public appreciates an artist. Another meaning is as an ethnic group. In either of these meanings, "people" is singular, and its plural is "peoples".

"People" is also, and more commonly, a plural noun, in the sense of "persons" (like "six people", "the people on the street"). In that sense, it should be replaced by the latter word if used with numerals. Definitely, one should not say "one/a people" for "one/a person".

possess. A formal equivalent of "have" or "own", better replaced by them when possible.

She possesses great courage.

She has great courage (is very brave).

He was the fortunate possessor of...

He was lucky enough to own...

practise and practice. In UK (not US) "practise" is the verb of "practice" (like "advise" and "advice"). Thus "a practised speaker" but "a speaker with a lot of practice" and "in practice".

problem. Overused and often ambiguous in mathematical writing. It refers to both a concrete mathematical task, and a difficulty in general. Consider the following examples:

When solving this problem we encountered a number of problems.	We describe special problems arising when solving this type of equation.
--	--

In the left sentence, the first "problem" can be replaced by "task", and the second by "difficulties". The right sentence leaves ambiguous what nature of problem (concrete task, or general difficulty) is meant.

reason. In the phrase "the reason for ... is because", the word "because" should be replaced by "that" (or "the fact that" or "the circumstance that" when "... is long), for "because" means "for the reason that". Often phrases with "reason" can be rewritten more straightly:

The reason for the lack of convergence is that there is a repelling fixpoint.	There is no convergence because of a repelling fixpoint.
---	--

regretful and regrettable. "Regrettable" means deserving regret (often a thing); "regretful" means feeling or expressing regret (always a person).

I feel regretful for this mistake.	It is a regrettable mistake.
------------------------------------	------------------------------

relate. A transitive verb, which therefore requires objects. Means "put into relation to", "establish a relationship between". Best used as "relate A to/and B" where A and B have some logical sense (principle, theory, property). Avoid for personal relationships.

I relate well to Janet.	I and Janet have a lot in common.
-------------------------	-----------------------------------

respective(ly). The adjective is often redundant.

The papers are listed in alphabetical order of their respective authors.	The papers are listed in alphabetical order of their authors.
--	---

The adverb can often also be avoided by rewriting, when expressing similar relationships.

The 1000m and 2000m races were won by Smith and Jones, respectively.	The 1000m race was won by Smith, the 2000m by Jones.
--	--

However, "respectively" is useful when the relationship is complicated and its repetition is awkward. This can in particular occur in a mathematical argument. The following is an example where the use of "respectively" is justified.

In order to show regularity, we remark first that the positive semidefiniteness of the matrices M and N follows from the properties (2) and (3), respectively.

Here are three alternatives obtained by trying to rewrite this sentence.

In order to show regularity, we remark first that the positive semidefiniteness of M follows from the property (2), and the positive semidefiniteness of N from the property (3).

In order to show regularity, we remark first that the positive semidefiniteness of the matrices M and N follows from the properties (2) and (3).

In order to show regularity, we remark first that the positive semidefiniteness of M follows from the property (2), and the one of N from the property (3).

In this case, neither of the three options is fine. First, it is correct, but less elegant to write the relationship out twice. Next, one should not simply omit "respectively", for "A implies B and C implies D" says something different from "A and C implies B and D". Finally, the use of "one" as reference to "positive semidefiniteness" creates an ambiguity with a reference to "regularity".

In any event, one must still make sure it is clear what separation "respectively" refers to. In:

The symmetry and regularity of the matrices M and N follow from the properties (2) and (3), respectively.

it must be clear whether (2) settles both properties for the one matrix and (3) for the other, or (2) deals with the first property (for both matrices) and (3) with the second.

satisfy and verify. There seems a common (and sloppy) use of the former for the latter. In mathematics, "verify" means to establish the truth of a statement or equality, and is a synonym for (though less colloquial than and so preferable to) "check". It is thus the mathematician who verifies. "Satisfy" means for a quantity to make an equation true when substituted for some variable therein, or for an object to have certain properties. Thus the set $\mathbb{R}^3$ does not verify the axioms of a vector space. Neither does a value verify an equation, and in the same way the author does not satisfy it.

We have now to verify that x is a maximal point.

We have now to show that x satisfies the sufficient conditions for being a maximal point.

For a person "satisfied that..." means to be convinced that, and "satisfied with..." to believe that something is sufficient/ good.

significant. As a qualifier (similar to "important", "interesting", etc.), to use sparingly in scientific writing. In statistics beware of confusion with its mathematical meaning.

so. As an intensifier ("so good", "so warm"), avoid in writing. Don't overuse as a conjunction replacing "thus" and "therefore".

As a demonstrative pronoun for way of doing something, better replace, e.g. by "this way", in strict writing; otherwise a confusion with some of its other functions is possible. In:

It is so easy to see that...

"so" could, in principle, mean any of "very", "therefore", or "this way". Note that even between "therefore" and "this way" there is a difference: the first indicates an immediately clear consequence, whereas the second a method, where possibly some details are skipped and have to be worked out before the conclusion follows.

split verb forms. This refers to compound verb forms, where an adverb is put before the final infinitive or participle. This construction has historical origin, and it should be used nowadays only if a particular emphasis on the adverb is intended or necessary: "I have been continuously asking for it."

state. As a verb, do not simply use as a replacement for "say". "State" means that the content is expressed fully and clearly, like a mathematical theorem.

3.4.7 T-W

"Than" is a comparative preposition, "then" a temporal pronoun or conjunction. "Than" is always preceded by some adjective in comparative form, or "other". "Different than" should be "different from" or "other than"; "something else than" should be "something else but".

Examine a sentence with "than" to make sure no essential words are missing:

I am probably closer to my mother than my father. (ambiguous: myself to my father, or my father to my mother?)	I am probably closer to my mother than to my father.
--	--

It looked more like a squid than an octopus.	It looked more like a squid than like an octopus.
--	---

the foreseeable future. A fuzzy cliché. How much foreseeable, by whom?

the truth is..., the fact is... A weak sentence beginning. Often it is much better to state the truth directly, when you know it.

they. He or She. "They" should not be used when, in a subordinate clause, referring to a subject named "every one", "everybody", "someone", "anybody", etc.

Every one of us knows they are not perfect.	Every one of us knows he is not perfect.
Everyone in the community, whether they are a member of the Association or not, is invited to attend.	Everyone in the community, whether he is a member of the Association or not, is invited to attend.

Sometimes, "they" is favored in order to avoid "he" as a gender-discriminatively perceived term, or the repetition of the awkward "he or she" (resp., "his or her"). There are, though, often other ways to avoid this problem. The following example shows three of them.

Use the plural rather than the singular.

The author must address his readers' concerns.	Authors must address their readers' concerns.
--	---

Eliminate the pronoun.

The author must address his readers' concerns.	The author must address readers' concerns.
--	--

Substitute the second person for the third.

The author must address his readers' concerns.	As an author, you must address your readers' concerns.
--	--

tired of and tired from. Although the separation is not definite, one is usually tired of something tiresome (annoying, bothersome), while tired from something tiring (painstaking, exhausting).

transpire. From the Latin "breathe across or through". Should be used for "become known", "leak through", not for "happen", "come to pass".

try. Takes the infinitive with "to" (for an isolated or occasional attempt) or the present participle (for a continuous or often repeated attempt). "Try and" is colloquial and to replace by "try to" in written English.

unique means "one of a kind", "without like or equal". As such, qualification of uniqueness ("so unique", "most unique", "very unique") is meaningless. If qualification is intended, likely "distinctive" or "special" is meant, and should be used, instead:

the most distinctive feature	a very special moment
------------------------------	-----------------------

very. A qualifier, to use sparsingly in formal writing. When possible, use instead words strong in themselves.

which, who(m) and that. As a general rule, "which" informs and does not restrict, or restricts from a relatively small group. In contrast, "that" defines and restricts (out of a larger group). Thus which-clauses often have commas, that-clauses almost always have none (see §3.1.5).

Consider the matrix, which is positive definite.	(one matrix, and it is positive definite)
Consider the matrix which is positive definite.	(among several matrices we have, look at the positive definite one)
Consider the matrix that is positive definite.	(an alternative to the previous example, possibly indicating that we choose the matrix from a larger group)
Consider a matrix that is positive definite.	(among all possible matrices, look at a positive definite one)

This rule seems not entirely binding, and the punctuation around the clause is important in deciding about restrictiveness and non-restrictiveness.

Sometimes an interchange between "which" and "that" can be useful in either direction to avoid repetition. Often, though, there are more elegant rewritings.

This approach is similar to that which we used.	(but better: "This approach is similar to the one (that) we used.")
This is a case which that method leaves unsolved.	(but better: "This is a case that the previous method leaves unsolved.")

"Which" and "that" should not refer to persons. "The teacher that I saw yesterday" is "the teacher (whom) I saw yesterday".

Be aware that omitting "which" or "who(m)" in non-restrictive clauses usually alters the scope the clause refers to:

The teacher whom I saw yesterday was new.	The teacher I saw yesterday was new.
The teacher, whom I saw yesterday, was new.	The teacher, I saw yesterday, was new.

In top pair of sentences omitting a restrictive "whom" preserves the sense, and even enhances fluency. In the bottom pair, omitting the non-restrictive "whom" changes the scope of what I saw yesterday from the teacher to the circumstance that he was new.

while. Avoid the indiscriminate use of this word for "and", "but", "although". (I confess: I'm, too, sick of the word!) When substituting "and" or "but" from

the desire to vary the connective, or from uncertainty which of the two is more appropriate, use a ”;”.

The director had his office on the top floor, while the lower floors were used by ordinary employees.	The director had his office on the top floor; the lower floors were used by ordinary employees.
---	---

The original meaning of ”while” is ”during the time that” or ”at the same time”. Replacing some of these for ”while” gives a good test whether the word is properly used.

If the contrast between two things is to be emphasized (and not their simultaneity), a word that does often better is ”whereas”. It should be put in the beginning of the second compared clause, no matter which of the two is introduced by ”while”. Consider the following:

While the nights are chilly, during the day we may have over 30°C.	The nights are chilly, whereas during the day we may have over 30°C.
--	--

In the following example, ”while” is used correctly (since it indicates two simultaneous lines of thought):

While I appreciate his work, I still have some points of criticism.

Note that when the following clause contains no verb (or gerund), ”while” should be replaced by ”during”. However, this rule does not apply if an implicit ”being” is omitted:

During a vacation, many people go on a trip.	(at the time of a vacation)
--	-----------------------------

While (being) a vacation, this was still a busy period.	(this was a vacation, but still it was a busy period)
---	---

-wise. A pseudosuffix that can be added to (but not only to) virtually every noun to turn it into an adverb. Chiefly useful when meaning ”in the manner of”, like in ”otherwise”, ”likewise”, ”pairwise”, ”clockwise”. But many wild and fancy creations like ”bottlewise”, ”pricewise”, ”marriagewise” are superfluous.

would. When expressing past habitual or repeated action, unnecessary (except maybe in some literaric work) in the presence of frequency-indicating expressions like ”every Sunday”, ”once a year”, ”often”. In that case, replace by the past tense. Thus write:

He would wake up early and prepare his own breakfast before going to work.	Every day he woke up early and prepared his own breakfast before going to work.
He would visit the old house.	Once a year he visited the old house.

In narrative writing, when using "would" sentences together with past tense sentences to describe a habit, always indicate the transition from the general to the particular. That is, make clear what actions constituted a general habit, and what actions (on the background of this habitual doing) took place on a specific day, moment, or period.

3.5 When English is a foreign language

Here some problems are described that occur in particular when English is not a native language. They include usage of words (and serve as a completion of §3.4) and expressions.

3.5.1 Articles

a, an, the. A great problem for many languages (incl. Japanese, Korean and Russian, among those I dealt with), which have no articulation.

Explanation is complicated. Probably usage can be accustomed to only by reading or listening experience. Here are some rules of thumb.

- "a(n)" is *indefinite* and often conveys in a weaker form the meaning of "some" or "a certain". If some of these fits well, then "a" is usually better than "the".
- "the" is *definite* and often conveys in a weaker form the meaning of "this/that".
- An article is not needed for the indefinite form of plural nouns and for nouns preceded by possessives. The latter mean the "'(s)" forms and possessive pronouns, including the relative pronoun "whose".
- A singular noun without article conveys the sense of an abstract or general notion, or an uncountable quantity.

In the jar there is jam.

Life is hard.

Compare the following examples:

Mathematics is interesting.

The mathematics is interesting.

Indefinite integrals may not have closed form solutions.

The indefinite integrals may not have closed form solutions.

If one uses "the" in the last two examples, the meaning is a little different. "The mathematics" means something like "the mathematics (the mathematical work) in the paper I am currently reading". And "the indefinite integrals" mean those particular ones I have to deal with.

- Do not omit the definite article when a plural noun is countable and is not meant in an abstract sense:

Derivatives of this function are positive.

The derivatives of this function are positive.

- An article is better omitted when a noun comes with a numeral or is a title of a proper name:

Using the Theorem 3	Using Theorem 3
In the Chapter XII	In Chapter XII
The president Smith said...	President Smith said...
The professor Smith said...	Professor Smith said...

Use singular nouns without article only if you are relatively sure. In general it seems that a mistake more likely occurs when an article is omitted than when one is used.

- If a noun precedes a label of a statement or a mathematical expression, the article is optional.

Using the equation (3.2),...	Using equation (3.2),...
The triangle ABC is right-angled.	Triangle ABC is right-angled.

The right sentences are also fine, since saying "using (3.2)" or " ABC is..." is correct, and "equation" and "triangle" work as attributes.

3.5.2 Modal verbs

can, may, and must. The usage of these modal verbs is very different from usual verbs.

- present. In the present tense, the verbs can be circumscribed by "be able to", "be allowed to", and "have to". "May" has, in the present tense, a secondary meaning like "will possibly decide to", "(will) have opportunity to" (for people), or "there is the possibility that" (for things).
- future. Only the circumscribed phrases allow one to build a future. The secondary meaning of "may" refers to an upcoming state and needs no grammatical future. Be sure it is clear that the intended sense is properly conveyed; in case replace by some of the circumscribing expressions.
- past. The grammatical past forms "could" and "might" serve rather badly for building that tense.
 - "Could" often indicates, like "may", a present or future possibility/opportunity instead of a past capability, and should be used for the past tense only if this intention is clear.
 - "Might" always indicates an even more hypothetical opportunity or possibility than "may" and "could", and never means "was allowed to". Thus the use of the circumscription past forms is recommended for "can" and necessary for "may".

- "Must" has no (even grammatical) past form, and the use of "had to" for past is imperative.
- Note that "might", despite being grammatically past, still expresses the present tense, and if the past is needed, phrases like "had some opportunity to" or "was possibly considering to" should be used.

– interrogative.

- In questions, if the subject is "I", almost always all of "can/could" and "may/might" mean, with different degree of politeness and expectation, "be allowed to". A rare exception is:

I asked myself: can I solve this problem?

- In the second or third person, "can" and "could" can mean "be able to", "be so kind to", or "have opportunity to", and which of these should be either clear, or this should be achieved by rephrasing.
- "May" and "might" in questions in the second or third person should never be used for "is/are/was/were allowed to", but always mean "have (some) opportunity to" or "be so kind to".
- For "must", again, one can build an interrogative form only via "have to".
- All of "can/could" and "may/might" indicate the present tense, except for "could" at places where this is clearly visible from the context. If the past or future is to be built (explicitly), use the alternative phrases.

3.5.3 Gerunds of verbs

-ing. A way of making nouns out of verbs. Many of these nouns are now well-established, sometimes with a meaning slightly different from just "the action of doing that".

In standard constructions like "the purpose of...", "there is no sense in..." such gerunds usually look fine. Still not all of these creations are to be used unrestrictedly like every other noun.

In particular, their plural is highly suspicious. It should not be used as an abbreviation of "acts or activities of doing that". Only in a handful of cases do plural forms have a meaning, and this meaning is often special.

The word has several meanings.

After several readings I understood the paper.

Take care of your belongings!

proceedings of a conference

human beings

earnings

savings

paintings

The word "doings" seems to be also acceptable, but has (in opposition to "deeds") a rather dubious subverse.

The word "the following" is very seldom meant as "the action of following something" (where a good substitute would be "pursuit"). It is more commonly used as an abbreviation of "the following thing" (where "thing" can be order, speech, statement, etc., depending on the context). As such it has no plural; instead of "the followings", one should write "the following statements, properties, conditions, etc."

Often other nouns stand better for gerunds. For example, there are hardly occurrences of "managing" that could not be substituted by "management", or "racing" by "race". "Acting" can be replaced, depending on the context, by "performance", "conduct", "action" or "function" (although "act" refers to a completed action, while "acting" more often to an ongoing one). Also, "seeing" or "watching" likely means some of "encounter", "spectatorship", "observation", "eyesight", "insight", or "perception".

More elegant reformulations are also possible by slightly larger changes. Here are two instances:

His making fun of people was not received well by his colleagues.	His insulting jokes were not received well by his colleagues. Making fun of people did not earn him much sympathy among colleagues.
His watching a movie was a rare event.	His attendance at the movie theatre was a rare event. or simply: He seldom came to watch a movie.

3.5.4 Plural forms

The following describes several ways of building plural (see [3]).

– Nouns ending on "ch", "s", "sh", "x" usually build a plural form by adding an "es" to the singular form:

glass-glasses, dish-dishes, ditch-ditches, wish-wishes, coach-coaches, kiss-kisses, tax-taxes

– The same applies to nouns ending on "o" which is preceded by a consonant:

hero-heroes, tornado-tornados-tornadoes (can end in either "s" or "es"),
potato-potatoes, tomato-tomatoes
but: radio-radios, ratio-ratios, duo-duos

– Nouns that end on a *single* "f" or "fe" usually build plural by changing the "f" into a "v" and adding "es" or "s"

half-halves, leaf-leaves, calf-calves, thief-thieves, grief-grieves, belief-believes

also: knife-knives, wife-wives, life-lives

Note: some just add an "s". This is in particular true for nouns ending on "ff".

proof-proofs, muff-muffs, playoff-playoffs

Some can do either:

dwarf-dwarfs-dwarves, hoof-hoofs-hooves.

– Nouns ending on "y" preceded by a consonant often have plural formed by dropping the "y" and adding "ies":

cherry-cherries, lady-ladies, story-stories, party-parties, diary-diaries

but: key-keys, monkey-monkeys, guy-guys (vocal before the "y")

– Many words of Greek origin ending on "-on" build plural by turning the "-on" into "-a":

criterion-criteria, analogon-analoga, polyhedron-polyhedra

but: skeleton-skeletons, hexagon-hexagons (regular plural)

This rule does not apply to the (much more numerous) nouns ending on "-tion" and "-sion". Another class of words on -on with regular plural are the elementary particle names.

– Most Latin words ending on "-um" turn this ending into an "-a" in plural:

forum-fora, equilibrium-equilibria, curriculum-curricula, scholium-scholia, momentum-momenta, stratum-strata

– Most Latin words ending on "-us" keep the Latin plural turning this ending into an "-i". In certain cases the English plural form using "-es" is also possible. In a few cases, only one of the two forms exists:

focus-foci-focuses, cactus-cacti-cactus(es), radius-radii-radiuses,

rhombus-rhombi-rhombuses, but:

locus-loci, stimulus-stimuli (no -es form),

campus-campuses (no -i form),

genus-genera-genuses (irregular)

In general, in formal writing the Latin plural of scientific words reads better.

- A few irregular plural nouns are formed by simply changing the vowel sound of the singular form:

foot-feet, goose-geese, louse-lice, man-men, mouse-mice, tooth-teeth,
man-men, woman-women (and other derivatives of -man)

- These irregular plural nouns obey no particular pattern:

child-children, ox-oxen, atlas-atlantes

- Some nouns have plural forms identical to the singular form (some have also the regular plural form):

deer, fish(es), moose, offspring, series, sheep, species

- These plural nouns have no singular form:

annals, billiards, thanks, belongings, clothes, congratulations, earnings,
eyeglasses, goods, groceries, jeans, pajamas, pants, proceedings, scissors,
shorts, sunglasses, savings, surroundings, tropics, trousers

Here are some particular cases.

data. A plural word, with no singular form, and best used with a plural verb form. Seems, though, to be gradually accepted in singular. Do not confuse with "date", and use "figures" only for statistical, not computational or experimental, data. Similar with media and *news*.

brains. In the sense of "intellect" (not human organs) always plural. Similar with looks (appearance).

clothes is a general term for cloth articles of personal and household use, usually such that are sold in some shop (same as "clothing"), or that can be washed. It is always plural. It is not the same as, and should not be used for the "cloths" (without "e") I have, I like or I wear.

glasses. In the sense of "what you wear to see better" no singular (similarly "eye-/sunglasses"). The singular is a lens, monocle (old-fashioned reading tool), or eyepiece (ocular of a micro-/telescope).

manners. In the sense of "social behavior" (not the way or method of doing something) always plural.

Several nouns which are used in plural in other languages are singular in English. They often refer to the totality of objects or persons of some type, or to an unmeasurable quantity. For other nouns there is a difference in meaning between singular and plural.

economy and economics. "Economy" has in the sense of "to consume less of something" no plural. In the sense of "production, trade, etc., of a country", its

plural is often used as a substitute for the countries whose economy is meant: "developing economies". "Economics" is the science for studying the economy of countries, and has no plural.

information. Always singular.

interest. The singular is used for "liking or inclination to do a certain thing" and for "profit rate" on a certain amount of money. When *one* thing is the object of attention, use singular, even if many people are interested in it. The plural should be used if one or several people are interested in *many* things, or in a political sense (demands, conditions).

Your interest in our program is appreciated.	He had diverse interests: reading, sports, travelling.
Because of great public interest, an extra performance was scheduled.	The negotiations respected the interests of the U.S.

material. A noun which, depending on the nuance of its meaning, may be in plural or not. No plural in the sense of "what you use to build or construct some (particular) thing". This does not necessarily need to be physically, but can also be logically, in the sense of "content". Always plural when meaning products needed for some general activity, mostly house construction or renovation, or things (books, stationery, writing pads) you use to learn or to do paperwork.

Take your learning materials.	The material in the book is interesting.
roofing materials	What material do you make this furniture from?

mathematics, physics, politics. Always singular.

staff. Staff is the group of all employees; as such it should be singular. The plural "staffs" might be tolerable if employees of several institutions (or administrative units) are meant, but even there the singular sounds (to me) better.

stuff. A singular word. The plural word is "things" or "items". Similar with furniture (plural: "pieces of furniture") and *equipment* (plural: "devices, cloths, tools", depending on the context).

3.5.5 ing-form vs. to+infinitive in compound verbs

The second verb in a combination often occurs in its ing-form or as to+infinitive. There is no absolutely clear rule when to use what, but a general guideline is that -ing refers to a general, continuous, and/or repeated action, while to+infinitive stands for a momentary and/or one-time action.

The following verbs usually go only with one of the forms.

- The verbs *interrupt*, *keep/continue*, *repeat*, *start/begin*, *stop/finish*, *quit/give up*, *postpone*, *extend* (as first verb) refer to the timely or repetative course of the action (of the second verb), and thus should be used with -ing.
- The verbs related to probation *attempt/try*, *fail*, *succeed/manage* (in the sense of succeed), and inclination or will *want/wish/desire*, *choose*, *decide*, *consent/agree*, *decline/refuse* usually indicate a momentary state, and thus should be used with to+infinitive.
- The verbs *supervise* and *manage* (in that sense) indicate a continuous action, and should be use with -ing. In contrast, *suggest/advice* are one-time, and should be used with to+infinitive. However, *recommend* (despite a momentary action) is used with -ing; similarly *admit* and *deny*.
- Verbs relating to state of mind like *think*, *believe*, *plan*, *hope*, despite their continous nature, need 'to'; however, *consider* needs the more suggestive -ing.

Some verbs can be used with both constructs, and there the dichotomy of usage is quite close to the one outlined above.

- The verbs of taste *prefer/like/love* can be used with to+infinitive or -ing depending on the continuity/repetativeness of the action. However, *dislike/hate* refer to an action in general (as opposted to 'don't like' for a momentary state), and thus should be used with -ing. Similarly, *enjoy* needs -ing, for it refers to a state during a general or continuous action.
- The verbs *allow/permit* and *forbid/prohibit* allow both choices based on the same criterion. Often (though the separation is not clear), the latter verb in the pairs refers to the general action (and uses -ing), while the former verb to the momentary one (and uses 'to').
- The verb *learn* usually uses 'to' even if the aquisition process is continuous. However, at the explicit indication of a period or frequency, still -ing is better.
- *Propose* seems to be fine with both constructions, and there seems not much difference in meaning.

In some cases the constructs can be iterated, linking three (or more) verbs. The form of each next verb depends on the nature of the prececing one, regardless of the others.

3.5.6 Some misspellings

Here are a number of misspellings of homophonous or almost homophonous words, which occur more frequently among non-native speakers. They are an addition to §3.4.1.

accept (agree to receive)	except (but, excluding)
adapt (modify)	adopt (take up, accept)
advice (noun)	advise (verb)
complement (the rest)	compliment (a flattering remark)
device (noun: scheme)	devise (verb: invent)
insight (a fact you understand)	inside (the inner part)
sign (+ or -)	sine ($\sin x$)
staff (employees)	stuff (things)
two	to/too

"Two" is a number, "to" is preceding a verb form, or a preposition, "too" means either "overly"/"in excess" (in front of an adjective), or "also".

3.5.7 Some phrases

capable. Do not use with to-infinitive, instead with "of" + gerund:

capable to do	capable of doing
---------------	------------------

equation and equality. Notice the difference between both. An "equality" $A = B$ holds for all values of the indeterminates occurring in A and B . In an "equation" $A = B$ proper values of the indeterminates in A and B are sought which make the statement correct.

possibility. It is not usual to say "possibility to". Use "possible to" or "can".

We have the possibility to obtain the solution using the Gauss-Jordan algorithm.	It is possible to obtain the...
--	---------------------------------

or simply: We can obtain...

since and for. "Since" is used with a point of time in the past, "for" for a period of time (between that point and now);

The solution has been known for 10 years.	The solution has been known since 1989.
---	---

Sometimes "since" can be used also with an event or some phrase describing a

particular action, implicitly referring to the time this action was taken.

Since the opening of the museum...

Since the work of Jones... (Since the time Jones did his work...)

permit and allow. If you use with to-infinitive, requires an object. Alternatively, use passive voice.

This construction permits to examine...

This construction permits us to examine...

or: This construction permits the examination... (passive voice)

3.5.8 Collocations

These refer to words similar in meaning, which may be alternatively be used in ordinary language. In mathematics, though, some words have no precise meaning, while other have some, and a different one.

For example, "magnitude", "size", "area" and "dimension".

Similarly, "uniquely determined" is not "uniquely fixed" or "uniquely decided" in mathematics. "Weak convergence" has a mathematical meaning, while "unstable convergence" or "feeble convergence" have none.

3.5.9 Idioms

These are phrases whose meanings are different from what the words inside would usually say. The best advice is, in the beginning, to avoid idioms, unless you are sure of their usage. Here are some examples.

End up. Reach a final state eventually.

It goes without saying. Is it so obvious that it does not need any mention.

On the other hand. From the other point of view. "Hand" can be replaced also by "hand side", but not (as in other languages) by "side" alone. This phrase introduces some opposition to the previous speech. Some people demand that a sentence with "On the other hand (side)" should be preceded by one explicitly containing "on the one hand (side)", and that both sentences should not be too far apart. I do not deem such strictness necessary, but it should indeed be made clear what the speech after "On the other hand" opposes to.

On the whole. In general, ignoring minor details.

Rule of thumb. A rule based on experience and estimation rather than rigorous calculation or reasoning.

Start from scratch. To start from the beginning with no help.

Trial and error. Attempting to achieve a goal by trying different possibilities and to find one that works.

3.6 An approach to style

4 Mathematical writing

4.1 What is Mathematical Writing?

1. Writing where mathematics is used as a primary means for expression, deduction, or problem solving.
2. Math writing combines two languages (natural and math)
 - Natural language is rich and allows for ambiguity
 - Math language is concise and must be unambiguous
3. Math writing requires slow reading
 - Often expresses complex ideas
 - Often must be read and pondered several times
 - Often is used as reference
 - Usually must be read selectively and in pieces
4. Very hard to teach to others (“Effective exposition is not a teachable art. There is no useful recipe” Halmos)
5. reflects personal style

4.2 Rules of Writing

Basic rules, according to your intended interaction:

1. Have mercy on the reader,
2. Have mercy on the editor/publisher, and, last but not least,
0. Have mercy on the author (i.e., yourself and/or your coauthors)

According to their scope, we can distinguish (not exclusively) between

1. Local rules
 - Apply to a single sentence (e.g., sentence structure rules mathspeak rules, comma rules, etc), or
 - to multiple consecutive sentences (e.g., join or break sentences)
 - Are verifiable

2. Global rules (our focus in this talk)

- Apply to the entire document
- General style and writing strategy rules
- Those concerning linguistic organization are mostly non-verifiable (e.g., organize well, be clear and concise, etc) or at least controversial (what is a good reference, example?; how much is a good explanation?)
- Those concerning mathematics must be verifiable (Consistency rules, flow of arguments in proofs etc.)

EXAMPLES OF LOCAL RULES

1. Break up long sentences into simple ones
2. Mathspeak should be "readable"
 - BAD: "Let $k > 0$ be an integer."
 - GOOD: "Let k be a positive integer" or "Consider an integer $k > 0$."
 - BAD: "Let $x \in R^n$ be a vector."
 - GOOD: "Let x be a vector in R^n " or "Consider a vector $x \in R^n$."
3. Don't start a sentence with mathspeak.
 - BAD: Proposition: f is continuous.
 - GOOD: Proposition: The function f is continuous.
4. Use active voice ("we" is better than "one")
5. Minimize "strange" symbols within text
6. Make proper use of "very", "trivial", "easy", "nice", "fundamental", etc
7. Use abbreviations correctly (e.g., cf., i.e., etc.)
8. Comma rules
9. "Which" and "that" rules
10. ...

EXAMPLES OF GLOBAL RULES

1. Language rules/goals to strive for: precision, clarity, familiarity, conciseness, fluidity

2. Organizational rules (how to structure your work, how to edit, rewrite, proofread, etc)
3. Down with the irrelevant and the trivial (Halmos)
4. Honesty is the best policy (Halmos)
5. Defend your style (against copyeditors - Halmos)

Consistency rules (be boring creatively)

6. Use consistent notation and nomenclature
7. State results consistently
8. Don't underexplain - don't overexplain

Readability rules (make it easy for the reader)

9. Explain your goals and the status of all claims
10. Use suggestive references
11. Consider examples and counterexamples
12. Use visualization when possible
13. ...

4.3 General Structuring Rules

4.3.1 Ordering

1. Typically a paper consists of title, abstract, introduction, body, and list of references.
2. A strong title identifies the general area of the subject and its most distinctive features. It helps locating the content of the paper in the body of mathematics. Include well-known keywords
 - BAD: "Concerning a certain application of a theorem of J. Doe"

- GOOD: “Algebraic properties of bounded operators”

Title should not be too long (say, more than 12 words, as otherwise often misspelled, misquoted) and not serve as an inflated advertisement.

3. Abstract identifies the subject (a table of contents in a paragraph of prose); no general background material here
4. start introduction strong; explain your subject and results and how you’ll present them. A good place to state main theorem, give historical comments, and outline the structure of the paper.
5. body contains main material (motivational discussions, definitions, results, proofs)
6. main reasons for dividing the body into sections:
 - the division indicates the strategy of your presentation
 - it allows readers to quickly and easily find the information that interests them;
 - indicates a breakpoint in the readers’ lecture
7. It is often helpful to begin a new section of the paper with a summary of the general setting.
8. Consider the use of appendixes and preliminary sections for background or difficult/specialized material.
9. Some redundancy should be built into the paper so that one or two misprints cannot destroy the understandability. (“error-correcting codes”)
10. As a very general rule, the definitions should go before the results that they are used in (Basic rule 1).
11. Similarly, results should go before their proofs (so that a reader can better follow the proof knowing where you are aiming). On the contrary,
12. introductory comments and motivation can be helpful to state before the results.

4.3.2 How to call statements?

1. Decide how to state results with the minimum of symbols.
2. A “theorem” is a major result, one of the main goals of the work. Use the term “theorem” sparingly.

3. Call a minor result a “proposition” if it is of independent interest.
4. Call a minor result a “corollary” if it follows with relatively little proof from a theorem, a proposition, or another corollary. It can be a consequence of the latter, or a generalization.
5. “Corollary” is considered less important than “proposition”.
6. Call a subsidiary statement a “lemma” if it is used in the proof of a theorem, a proposition, or another lemma.
7. A formal definition should simply introduce some terminology or notation; there should be no accompanying discussion of the new terms or symbols.
8. A formal remark should be a brief comment made in passing; the main discussion should be logically independent of the content of the remark.
9. Often it is better to embed definitions and remarks into the general discussion rather than setting them apart formally.
10. If your paper raises a natural question, and you don’t know the answer, by all means say so! This may turn out to be more interesting than the theorems that you prove.
11. Conversely, refrain from making “conjectures” too hastily. Use instead the words “question” or “problem”. Remember that a good “question” should be answerable by “yes” or “no”. To ask “under what conditions does A hold” is often not a question worth setting off.

4.3.3 Definitions

1. All basic definitions should be given if they are not a standard part of the literature. It is perhaps best to err on the side of making life easier on the reader by including a bit too much as opposed to too little (Basic rule 1).
2. If a standard symbol is used without being defined formally, it might help to write its meaning at the first (and a few other) occurrences. For example, if “ $\rho(A)$ ” is used, and regarded standard, for the spectral radius of a matrix A , then writing a few times “the spectral radius $\rho(A)$ ” instead of “ $\rho(A)$ ” will clarify things to an uncertain reader.
3. Recalling in the above way the meaning of a term can be useful also for defined terms, which have not been used on a number of previous pages.

4. A non-standard, author-coined, definition should be given only if the defined symbol or word is used in the following often enough to justify its prominence. Otherwise, the meaning should be written out at the few occurrences.
5. Ideally, a definition should be made where the term defined is first used. (Otherwise, a reader may lose concentration, or even interest, in referring back.) Alternatively, standard definitions can be grouped, often without being set off formally, into a preliminaries section.
6. In set off definitions it is usually enough to write "if" for "if and only if" (or "iff") characterizing the condition defining the new object.
7. Highlight a term being defined, usually by italics. (If the definition is not set off, one may otherwise confuse it with a result.) Alternatively, use a phrase like

A graph is said to be connected, if...	A graph is defined to be connected, if...
---	--

8. If a term is defined by means of an equality, you might use a special symbol like ":@" or " $\stackrel{\text{def}}{=}$ " to set the definition apart from an identity (this symbol should then be used consistently). Alternatively write "We set...", "We define..."

4.3.4 How to write proofs?

1. Emphasize the structure of the proof, difficulty of each step, and key ideas briefly in advance. Phrases that help in this regard:

The aim/idea is to...
 Our first goal is to show that...
 The difficulty now is to find...
 ...is the key relation.
 The essential observation is that...

2. Prefer a conceptual proof to a computational one; ideas are easier to communicate, understand, appreciate and remember. *The ideas are the mathematics.* Try first to formulate the ideas in the proof without computations.
3. Omit the details of purely routine computations and arguments – ones with no unexpected tricks and no new ideas. A page of computations without

any writing or explanation contains no mathematics. Merely state starting point, procedure and outcome. Phrases that help:

It is easy/simple/straightforward to show that...

Some tedious manipulation yields...

After a long but straightforward calculation, we obtain...

After two applications of... we find...

An argument similar to the one used in ... shows that...

4. Strive to keep the reader informed where you stand in the proof:

First we establish that...

So far we have shown that...

Our task is now to...

It remains to show that...

Finally, we have to show that...

We can apply now...

5. Read your writing, including all of the equations, out loud. Your ears can often pick out sentence fragments and grammatical errors better than your eyes.
6. Avoid long sentences and extremely long (or short) paragraphs.
7. Loosen long paragraphs (in particular in proofs, where they are hard to follow by the reader) by inserting displayed formulas, or by breaking such paragraphs into smaller ones.
8. Avoid long sequences of equations (or inequalities), without explaining the reader what goes on in between behind these equations (inequalities). Interject verbal comments between the equations, but not inside them (“two-column-style”).
9. Write the proof forward; that is, avoid unnaturally looking arguments, notations etc., which will be justified only by subsequent discussion (see the “notations” section below for an example). This is more graspable and rememberable.
10. Beware of any proof by contradiction; often there’s a simpler direct argument.

4.3.5 Format

1. Whatever format or style you choose to adopt, especially if it deviates from the publisher’s style, make sure that it is consistent. This is mostly a difficulty with books (Basic rule 2).

- Strive for consistent appearance – margins, spacing, layout of definitions, proofs, theorems; references. For example
 - If you leave a blank page at the end of one chapter so the next one can start on an odd-numbered page, then make sure you always do so.
2. Strive for parallelism – express similar concepts in a similar way. This can be relevant on many levels:
 - Notation. If we write "A special case of $P(x) = \prod_{i=1}^n (x - x_i)$ is $x^2 + ax + b = (x - x_2)(x - x_1)$ ", then the second formula may confuse the reader.
 - Similarly, if we write "if $a \leq b$ and $b \leq c$, then $c \geq a$ " instead of "... $a \leq c$ ", it looks less natural (though it is completely correct). In either example the reader may be tripped up to seek the reason why the less natural variant was chosen.
 - If an expression plays a similar role, or looks similarly to another, either display both, or none.
 - Formulate similarly theorems containing similar statements; likewise for analogous definitions. The analogy between facts and concepts should not be obscured for the mere sake of stylistic diversity. However, it is fine to say in Theorem 2: "Under the same assumption as in Theorem 1..."
 3. Writing a paper or book entails making choices of what material is important and what can be skipped. ("Stop" Halmos.) It is impossible to cover all possible results and so the material needs to be covered in a well thought out manner. A paper or book should not be considered an opportunity for showing off (Basic rule 1).
 4. *Avoid rambling.* Define your exact goals in the beginning and keep them in mind throughout your exposition.
 5. Leave out what is unimportant. If it goes without saying, don't say it. Concise writing is simple and efficient, thus beautiful.
 6. On the other hand, don't leave out anything which is critical to the key ideas you are trying to explain.
 7. One should avoid giving the reader the impression that the subject matter can be mastered only with great pain. In fact, this is an ideal way to lose readers (or audiences!).
 8. Bad phrases:

- Statements of sexist, racial, ethnic discrimination (A note: The use of “he/him/his” in the following is not meant as a sexist subverse, just as a tool of saving space – as suggest also Strunk and White.)
 - Detailed personal discussion of the (un)importance of (including libelous statements about) others’ mathematical work. (There are, however, delicate issues concerning references discussed below.)
 - Abusive statements about readers, editors, referees etc.
 - Monetary claims (like “An explanation of this statement is offered for \$25.”)
9. In the list of references, give the full page numbers of each article appearing in a journal, a proceedings volume, or other collection; do not give the numbers of the particular pages cited in the text. (see also “Use suggestive references” section)
 10. After the paper is finished, it should be reread (and, perhaps, rewritten) from the reader’s point of view (Basic rule 1).
 11. A good way to begin is to use a standard classic of mathematical exposition (e.g., Bourbaki-Algebra, works by Serre, Atiyah or Milnor) as a basic model.
 12. *No set of rules can ever be perfect!* Avoid slavish adherence to any set of writing rules, including the rules enumerated here.

4.4 Language

4.4.1 Words vs symbols

1. Use symbols if expressing the idea in words is too cumbersome, or the exactness of a statement is particularly important.
2. Use words when they do not take up much more space than the symbols. Keep in mind that words are easier to grasp by a reader than symbols.
3. Explain in words symbols’ meaning if you think the reader might have difficulty grasping it.

Example

For a matrix M , we choose v to be an eigenvector to an eigenvalue of M of minimal modulus, such that v is closest to $(0, 0, \dots, 1)$.

For a matrix M , let $\lambda_i, i = 1, \dots, n$ be its eigenvalues, and let λ_i be permuted so that $|\lambda_1| = \dots = |\lambda_k| < |\lambda_{k+1}| \leq \dots \leq |\lambda_n|$. Among all vectors v with $Mv = \lambda_j v$ for some $j = 1, \dots, k$, choose one for which $\|v - (0, 0, \dots, 1)\|$ is minimal.

The notation of the second version makes in not only harder to read, but also longer. If the purpose is solely to specify v , and λ_i and their ordering is not needed later, the first variant clearly does the job much better.

Another example:

if both $\{x_k\}$ and $\{y_k\}$ have only finitely many zero terms, then neither of $\{x_k\}$ or $\{y_k\}$ tends to 0. if $\#\{k : x_k = 0\} < \infty$ and $\#\{k : y_k = 0\} < \infty$, then $x_k \not\rightarrow 0$ and $y_k \not\rightarrow 0$

The first version is only little longer, though much smoother to read.

4.4.2 Punctuation and words around symbols

1. Observe the usual conventions in terms of spelling, punctuation, and the other basic elements of style. Most of what Strunk and White say on general English applies for mathematical English as well.
2. Linguistic errors are usually not fatal, but in excess distract (or even anger!) the reader.
3. Punctuate symbolic sentences as ordinary ones.
4. Do not overuse small punctuation marks as period and comma. They are easy to overlook and misinterpret. In particular
5. Words like "then", "and", or "or" should not be replaced by a comma. (The presence of "then" can never confuse; its absence can.)
 - It is bad to write "If $x = 2, y = 3, z = 4$ " meaning "If $x = 2$ and $y = 3$, then z is equal to 4" (or "If $x = 2$ and $y = 3$, then $z = 4$ "). Similarly "Since $p \in U$, we have $p \neq 0$ " is much better than "Since $p \in U, p \neq 0$ ".
6. Separate symbols in different formulas with words
 - Bad: Consider $S_q, q = 1, \dots, n$.

- Good: Consider S_q for $q = 1, \dots, n$.
 - “Assume that $a \in X$. X^* belongs to the class $C \dots$ ” is much more confusing than “Assume that $a \in X$. The operator X^* belongs to the class $C \dots$ ”
7. Readability. Write formulas so that they can be easily read out loud fluently (see also “General structuring rules” section).
- It is better to write “... we prove that $\frac{\zeta_{\mathbf{Q}}(2n)}{\pi^{2n}}$ belongs to $\mathbf{Q}$ ” (or “is rational”) instead of “... we prove that $\zeta_{\mathbf{Q}}(2n) \in \pi^{2n}\mathbf{Q}$ ”.
8. Avoid isolated relational symbols in text; replace them by words.
- Bad: “ f is continuous $\Rightarrow f$ is bounded”
 - Good: “ f is continuous, and therefore f is bounded”
 - Bad: “this number $<$ the expression in (1)”
 - Good: “this number is smaller than the expression in (1)”
9. Beware of using symbols to convey too much information all at once.
- Very bad: If $\Delta = b^2 - 4ac \geq 0$, then the roots are real.
 - Bad: If $\Delta = b^2 - 4ac$ is nonnegative, then the roots are real.
 - Good: Set $\Delta = b^2 - 4ac$. If $\Delta \geq 0$, then the roots are real.
10. Don’t set off by commas any symbol or formula used in text in apposition to a noun.
- Bad: If the discriminant, Δ , is nonnegative, then the roots are real.
 - Good: If the discriminant Δ is nonnegative, then the roots are real.
11. Precede a theorem, algorithm, and the like with a complete sentence.
- Bad: “We now have the following
Theorem 4-1. $H(x)$ is continuous. ”
 - Good: “We can now prove the following result.
Theorem 4-1. $H(x)$ is continuous.”
12. It is traditional to use “if” instead of “if and only if” in definitions.
13. Use the present – not the past – form.
- As an example of bad writing, we have: “We have proved that $f(x)$ was equal to $g(x) \dots$ ”. This is corrected to: “We have proved that $f(x)$ is equal to $g(x) \dots$ ”

14. You should not begin a sentence with a math symbol. This can confuse the printer as well as the reader (Basic rules 1 and 2), in particular if a previous sentence ends with mathspeak.

- As an example of such bad writing, we have: "... we want to prove the continuity of $f(x) = 2 \cos^2 x \cdot \sin x$. $\cos x$ being continuous..." This is corrected to: "... $f(x) = 2 \cos^2 x \cdot \sin x$. Since $\cos x$ is continuous..."

Similar caution is needed when clauses after a colon, semicolon or comma start with a math symbol, or an equation reference.

15. When using ellipsis, place punctuation symmetric around it: write " $1, 2, \dots, n$ " and " $x_1 + x_2 + \dots + x_n$ ", not " $1, 2, \dots n$ " and " $x_1 + x_2 + \dots x_n$ ". The centered ellipsis "..." better be used only to omit terms in compound expressions (and not to skip terms in an enumeration), and not between multiplication dots.
16. Mathspeak, in particular complex one, should not be (over)used as a substitute of a noun in grammatical constructs.

- Mathspeak plural forms should be used sparsingly, and only for very brief expressions. Things like

" $(\sum_i x_i y_i)$ -s"

are hardly acceptable. Similar constructs to avoid are:

- "There are many $(\sum_i x_i y_i)$ -like terms..."
- "... and $(\sum_i \frac{x_i}{y_i})$'s denominator is..."
- " $(\sum_i x_i y_i)$ -norm" (meaning the norm in L^p for $p = \sum_i x_i y_i$),
- "the $(\sum_i x_i y_i)$ -th term", etc.

- Note that using the 's in mathspeak plural forms can confuse with a possessive; thus use 's' or '-s'.
- Mathspeak plural forms can be often rewritten more smoothly (see also §3.2.7):

0s, 1s, x s zeros, one-s, occurrences of ' x '

There seems no need for a hyphen in "zeros" (unless there are zeros of a function or polynomial mentioned closely nearby). But beware of the grammatical meaning of "ones" (and "one's"), which creates unintended references in many contexts.

- In certain cases the plural ending after mathspeak (which is not a single number) can simply be omitted, as the plural can be inferred from the following verb. In other cases one can make the plural explicit

without ending by adding a quantifier like "all", "many", "several", "some of the", "those" before the mathspeak:

the x_i behave... This holds for all x_i .

(To the contrary, "one", "unique", etc., can unambiguously exclude the plural meaning.)

- Whatever ending is used for mathspeak plural, beware of confusions like between " x_i -s" and " $x_i - s$ " (hyphen vs. minus), " x_i s" and " $x_i s$ " (latter in the sense of " $x_i \cdot s$ ") and " x_i 's" and " x_i' s" (apostroph versus prime). Note that even if no ending is used, a reader may take some part of the mathspeak for one. To avoid these problems, use a different font for text and mathspeak (when typesetting), and write out the multiplicative ".".

4.4.3 Usage of words

1. Say what you mean. There are many ambiguities of meaning which can be introduced by uncaredful writing. Here are some common pitfalls:

- A pronoun normally refers to the first preceding noun, but sometimes it refers broadly to a preceding phrase, topic, or idea. Make sure the reference is immediately clear, especially with "it", "this", and "which". An example:

Condition 3 is not satisfied for this space, which is why we do not consider it further.

It is not clear whether the space or the condition is not considered further.

- Mathematical papers with a lot of "it" and "that" tend to be hard to read. It is often hard for the reader to see what "it" is referring to. If you are also having difficulty specifying what "it" is, then you may be having some difficulty with the mathematical ideas; you may need to think more about the ideas you are writing about.
- "The proof above", "the previous lemma", "as we noted before", etc., can also be inexact expressions and confuse the reader. Use them only if you are absolutely certain that such references can be correctly resolved.
- A similar problem can occur with "only":

This is conjectured only when x_i are real. This is only conjectured when x_i are real.

In the first sentence, the conjecture is made only in the case that x_i are real (with no statement about other cases). In the second sentence, it is implied that the statement was proved in other cases (previously discussed).

- Make sure what quantifiers refer to. The second phrase below can be uncarefully written to become the first, which has a different meaning.

A single n satisfies all equations. For each equation, there is a unique n satisfying it.

- Put emphatic words at the beginning of a sentence. These tend to give the reader the direction. Avoid starting sentences with weak phrases like "There is" or "There are". Also, often sentence openings like "It is clear that", "It is interesting to remark that" can be rewritten to make the sentence more fresh and lively.
- Usage of "we", "one", "I"
 - active voice ("we") is better than impartial voice ("one"); you want the reader to feel part of what you do in the paper
 - alternatively, one may use neutral voice (putting the content of the paper into subject and using passive verb: "The number p is found by multiplying q by r ." or imperative ("To find p , multiply q by r ."))
 - "we" means "the author and the reader" (or "the lecturer and the audience"). Usually acceptable, for example
 - "Using lemma 3 we can prove ..." or "The deduction of lemma 3 gives us the technique to prove ..."; however,
 - a single author to write "Our previous work on this problem was done in 1976." (did all the present readers look at that paper?) is improper, and "We thank our wife for the typing." (to how many people is she married?) is always bad.
 - "one" tends to mean "anyone" - proper in discussion of known material in the standard way
 - "I" points to personal sentiment or reaction; in particular in overuse, makes an arrogant impression, and so to use sparingly.
- Usage of "obvious": see later rules "Explain properly" (similarly "trivial" and "straightforward").
- Usage of "given": avoid when means nothing ("For any given p , we have...")
- Usage of "any": avoid since ambiguous – does it mean "some" (as can be in ordinary language: "Do you have any money?") or "every"? Replace by "every" or "each"; if necessary recast the sentence

7. Other troublesome words: “where”, “equivalent”, and “if... then... if... then”:
8. “Where” points usually to a lazy afterthought that can be mended by rephrasing the sentence. (“We have $a_n/b_n \rightarrow 0$, where b_n is...”)
9. “Equivalent” for theorems is logical nonsense. (A meaningful statement can be false; a theorem is always true; a “false theorem” does not exist; it is the equivalence of the *statements in the theorems*, and maybe the similarity of the difficulty of their proofs, that is actually meant.) Likewise a statement being equivalent to a theorem means logically only that it is true (it is its equivalence to the statement in the theorem that is actually meant).
10. “If p , then if q , then r ” (meaning “ $p \Rightarrow (q \Rightarrow r)$ ”) difficult to grasp; needs rewrite like “If p and q , then r ”, or “Under the assumption p , the condition q implies r .” (depending on what is to emphasize)
11. “contain” and “include”: helpful to distinguish. “ A contains B ” means “ $A \ni B$ ”, while “ A includes B ” means “ $A \supset B$ ”.
12. “sequence”: not to use for “an infinite collection”, for example:
 - BAD: “the union of a sequence of open sets is open” (the reader is pointed to the order of these sets, which is unnecessary and irrelevant)
 - GOOD: “the union of infinitely many (or of an infinite family of) open sets is open”; however
 - GOOD: “the uniform limit of a sequence of continuous functions is continuous” (here order of functions is relevant)
13. Always consider a word’s connotations (associated meanings); the wrong connotations can trip up your readers by suggesting unintended ideas.
14. Do not (over)use words like “capability”, “utilize”, and “implement”; they offer no precision, clarity, or continuity and smack of pseudo-intellectualism.
15. Don’t simply call a statement “important”, “interesting”, “fundamental”, or “remarkable”, but explain why it is so.
16. Beware of words like “interface”; they are precise in some contexts, yet imprecise and pretentious in others.
17. Beware of dangling participles.
 - It is wrong to write, “Solving the equation, the roots are real”, because we solve the equation, not “the roots”.

- Write “Solving the equation, we find that the roots are real”, or “With the equation solved, the roots are found real.”
18. One should avoid using abbreviations like “w.r.t.” (with respect to), “iff” (if and only if), and “w.l.o.g.” (without loss of generality). They simply do not look very nice (and “iff” is allegedly offensive to some readers; Rules 1 and 2).
 19. When words have alternative spelling, then stick to one. This concerns in particular the -ize and -ise variants of some verbs in US vs. UK. (Note also that some words do not have an -ize spelling: “advise”, “surprise”, “rise”.)
 20. Avoid contractions: “isn’t”, “it’s”, “let’s”, “don’t”, etc. And do not confuse “it’s” with “its” (see §3.1.3).
 21. Synonymic terms should not be mixed: a “prime factorization” should not be called “prime decomposition” elsewhere.
 22. Good Phrases to Use in Math Papers:
 - consequence: “Therefore” (also: “so”, “hence”, “accordingly”, “thus”, “it follows that”, “we see that”, “from this we get”, “consequently”, “then”)
 - assumptions and notation: “let”, “given” (however, see above!), “ M represents”, “assuming”, “where”, “ M stands for”
 - arguments: “show” (also: “demonstrate”, “prove”, “explain why”, “find”)
 - references: “(see the formula above)”, also: “(see (*)), this tells us that ...”
 - explanations: “notice that” (also: “note that”, “notice”, “recall”), “conversely”, “for example”, “in particular”
 - reason: “since” (also: “because”, “as”)
 - conditions: “if” (also: “whenever”, “provided that”, “when”)
 - emphasis: “actually”, “certainly”, “clearly”, “indeed”, “obviously”

4.4.4 More on punctuation

1. The citation is treated somewhat like a parenthetical remark within a sentence. Footnotes are not used; neither are the abbreviations “loc. cit.”, “op. cit.”, and “ibid.”
2. If the citation comes at the end of a sentence, put the period after the citation, not before the brackets or inside them.

3. Keep in mind that the display of mathspeak is not a figure, but an integral part of the sentence, and therefore needs punctuation.
4. Do not put punctuation marks (period, comma, quotation marks) and closing delimiters (parentheses, brackets, braces) as the first character of a line. Similarly with opening delimiters as the last character of a line.
5. Related to the previous rule (since line breaks usually occur at white space): Never put white space before, but always after punctuation marks and closing delimiters (unless they are followed by another character of these sorts). Exactly the opposite applies to opening delimiters. As a particular special case of this rule,
6. Put punctuation following a mathematical display just before end of the display, and not just after it (letting it stand in the beginning of the following text line).
7. Do not nest delimiters in ordinary text.
 - Replace delimiters by dashes or commas. If nesting is necessary,
 - Vary type of delimiters (parentheses within brackets within braces)
 - If nested delimiter groups begin (very unusual!) or end simultaneously, I still prefer “((” or ”))” than to wonder where to find a missing delimiter (or to guess it left out by mistake).
 - Exception: paper citation brackets and equation reference parentheses are fine in any delimiter group.
8. Do not start paragraphs with text in delimiters.

4.5 Use Consistent and Proper Notation

1. Choose a notational style and stick with it. Examples:
 - Use capitals for random variables, lower case for values
 - Use subscripts for sequences, superscripts for components
 - If one proof ends with a ”QED”, then they all should, etc.
2. Define every symbol, and do so before you start using it.
3. Try, as far as possible, to use symbols in such manner that they are read the same way:
 - BAD: “For $x \in A$, we have $f(x) \in B$ ” (“ $\in$ ” is once read as “in”, once as “is in”)

- GOOD: “If $x \in A$, then $f(x) \in B$ ” (“ $\in$ ” is read both times as “is in”)
4. Use suggestive/mnemonic notation. Examples: S for set, f for function, B for ball, ε for a small positive real number etc. In particular,
 5. Choose symbols that are consistent with mathematical convention. There are no hard and fast rules, but if you look for it, you will notice a remarkable consistency. For example,
 - real number variables are usually denoted by u, v, x , and y ;
 - real number constants are often denoted by a, b , and c ;
 - integers and natural number are typically denoted by i, j, k, n, m, p , and q ; and
 - functions are often named f, g , or h , with p and q often reserved for polynomial functions.
 - There are also “reserved” symbols like $i, e, \pi, \Gamma(x), \dots$, that often have a particular standard value.

The best way to learn the conventions is to pay attention to how symbols are used in lectures and in your reading.

6. Choose symbols consistent with your nomenclature: if you use “null-space”, call it $\text{null}(A)$, and “kernel” $\ker(A)$, not vice versa.
7. Use simple notation. Examples:
 - Try to avoid parenthesized indices: $x(m, n)$ vs x_{mn} , but
 - also avoid nested super/subscripts $x_{m_{n_l}}$. They are error-prone and difficult to read. They also lead the author to more computational and less conceptual proofs. (see “General structuring rules” section)
 - Similarly, beware of overusing accents. For example, $\hat{A}$ and $\tilde{A}$ can be difficult to distinguish in reading, and easy to confuse through misprinting.
 - avoid long expressions as super/subscripts: if u is long, $\exp(u)$ is better than e^u
 - Balance between the alternatives “ $\frac{a}{b}$ ” and “ $a^{-1}b$ ”, as well as “ $\sqrt{a}$ ” and “ $a^{1/2}$ ” (where a, b are arbitrary expressions). When a short, the former options seem better. However, for long a , latter options can be more readable, and only they allow for line breaks.

- Similarly balance between the options

$$\min_{\substack{X \text{ satisfies} \\ \text{condition 1} \\ \text{condition 2}}} f(X) \qquad \min \left\{ f(X) : \begin{array}{l} X \text{ satisfies} \\ \text{condition 1} \\ \text{and condition 2} \end{array} \right\}$$

depending on the complexity of the conditions on X and the quantity $f(X)$ minimized. In the second form, sometimes a vertical bar is used for the colon. Either character can be more useful in certain cases. But beware of things like

$$\min \{ |x| \mid x \mid \|y\| \}$$

(in the sense of "minimum of absolute value of x over x dividing norm of y ").

- The notation $\text{diag}(x_1, x_2, \dots, x_n)$ for a diagonal matrix (when clearly introduced) can save space in opposition to the matrix form.
- An example of simplifying a case-defined expression by giving meaning to extreme cases of notation:

$$a_{ij} = \begin{cases} 0 & \text{if } i < j \\ \frac{1}{c_j} & \text{if } i = j \\ \frac{1}{c_j} \prod_{k=j+1}^i \left(\frac{-d_k}{c_k} \right) & \text{if } i > j \end{cases}$$

$$a_{ij} = \begin{cases} 0 & \text{if } i < j \\ \frac{1}{c_j} \prod_{k=j+1}^i \left(\frac{-d_k}{c_k} \right) & \text{if } i \geq j \end{cases}$$

The lower form incorporates the (extreme) case $i = j$ into the one $i > j$ (using the standard convention that the empty product is 1).

- avoid underlining mathematical expressions
8. Sometimes there are several types of notation for the same object. For example, a matrix A can be given by its entries A_{ij} or as $A = [A_1 \ A_2 \ \dots \ A_n]$ by means of its column vectors. Choose the notation that is most suitable and helps explaining best your work.
 9. Avoid unnecessary notation – globally (if you aren't going to refer to something more than once or twice, don't introduce a symbol to describe it) and locally:
 - BAD: Let X be a compact subset of a space Y . If f is a continuous real-valued function over X , it attains a minimum over X .
 - BAD: A continuous real-valued function f attains a minimum over a compact set X .

- GOOD: A continuous real-valued function attains a minimum over a compact set.

This is true for statements even if notation is later used in proof:

- BAD: “Proposition: If $0 \leq \lim_n \alpha_n^{1/n} = \rho \leq 1$, then $\alpha_n \rightarrow 0$.
Proof. ... we use that ρ is...”
- GOOD: “Proposition: If $0 \leq \lim_n \alpha_n^{1/n} \leq 1$, then $\alpha_n \rightarrow 0$.
Proof. Let $\rho = \lim_n \alpha_n^{1/n}$...”

10. Avoid unnecessary delimiters: “the matrix $(A - \lambda I)$ is...”, or punctuation: if one writes “ $x_{1,2,3,2,1,3,2}$ ” as “ $x_{1232132}$ ”, then will be no problems, unless x exists (and makes sense in the context) with a single subscript which can be huge.

11. Avoid overlapping notation, i.e., the same symbol used (at least in brief succession) for different objects. One particular such situation is when variables are bound (to different values):

- BAD: “We have $P = 0.5(1.12)^t$, so that for $t = 2$ we have $P = 0.5(1.12)^2 = 0.6272$. Now if we want to know for which t we have $P = 2$, ...”
- GOOD: “We have $P(t) = 0.5(1.12)^t$, so putting $t = 2$ we obtain $0.5(1.12)^2 = 0.6272$. Now let t_2 be such that $P(t_2) = 2$. If we want to know t_2 , ...”

12. Avoid ambiguous notation. Instances are:

- Never change the name of an object in the middle of an argument.
- A fraction put in parentheses (for example as a base of a power) can confuse with the Legendre symbol.
- For a map f , the term f^{-1} can mean both the map-theoretic inverse, and the map $x \mapsto 1/f(x)$; similarly with other powers.
- Standard function names (sin, lim, gcd) should be typeset in roman (when variables are typeset in italic): it should be “ $\tan x$ ”, and not “ $\tan x$ ”, which may also be the product of four variables.
- Care is needed to avoid ambiguity of slashed fractions: is “ $1/12f'(1)$ ” to denote “ $\frac{1}{12}f'(1)$ ” or “ $\frac{1}{12f'(1)}$ ”?
- Roots are ambiguous over $\mathbb{C}$: while there is a well-defined meaning of $\sqrt[4]{x}$ when x is real (and non-negative), there are four values when x is complex. Similar with logarithms.

- Avoid the prime (') for a derivative, if a function has several variables to be partially differentiated along. Use notations like $f_x(x, y)$ (when the index is not ambiguous) or (better) $\frac{\partial f}{\partial x}(x, y)$. Note that even $\frac{d}{dx}$ is confusing (even if it contains the differentiated variable), because it is reserved for a one-variable derivative. Thus $\frac{df}{dx}(x, y)$ may suggest to the reader that y is a constant.
 - When multiplication is non-commutative, $\frac{a}{b}$ makes no sense. Neither should one write a/b for ab^{-1} , unless one introduces $b \backslash a$ as well, and points out the difference between both. Similarly, the iterated product $\prod$ should be made unambiguous. (If there is no unity, the empty product again makes no sense.)
13. The "quantifiers" should always be clear (Basic rule 1). Some examples to avoid:
- "We have $f(x) = g(x)$ ($x \in X$)."
 - What does the parenthesis mean? That $f(x) = g(x)$ for all $x \in X$, or, for some $x \in X$? Similarly,
 - does " $f \neq 0$ " mean $f(x) \neq 0$ for some x (then better is " $f \not\equiv 0$ ") or for all x ?
 - What does " $f_{t,u}(x, y) = O(g_{t,u}(x, y))$ " mean? Very often it means that t, u, y are fixed and x is allowed to vary. Quantifier problems are especially troublesome with "big O" notation.
14. The word "constant" is terribly ambiguous. It is important to make explicit exactly which variables the constant depends on.
15. You should explicitly state what all letters in your formulas represent in as precise a manner as possible. For instance:
- BAD: "Either n or $n + 1$ is even." What is n ? If $n = 8.5$ is the above statement true? A better way of stating this is:
 - GOOD: "For any integer n , either n or $n + 1$ is even."
16. Maintain distinction between functions and their values (see also above " $f \neq 0$ " example)
- BAD: the function $z^2 + 1$ is even
 - GOOD: the function $f(z) = z^2 + 1$ (or $z \mapsto z^2 + 1$) is even
17. Avoid logical symbolism ($\exists, \forall, \wedge, \vee$ etc.) – difficult to digest; however, for *really long* logical constructs it may be necessary (it is unambiguous, while verbal form can be misinterpreted)

18. Display mathspeak that

- goes over 1/4 of a line,
- is difficult to break between lines,
- needs special attention, e.g. because it contains the first occurrence of an important variable, or more generally if
- it will be referenced later.

19. Take care that delimiters in formulas pair up (that is, an opening delimiters has a matching closing one and vice versa).

- If a displayed formula inherits an extra parenthesis from a parenthesized clause in the surrounding sentence, put some words just before of after the display and put the unmatched delimiter (outside the display) into the text (see “Language” rules).
- Try to match as many as possible delimiters on the same line in formulas broken over several lines.
- Exception: case-by-case defined quantities may be given by an opening brace, without a matching closing one, *if* they are at the end of an equation, and not preceded by similar (or other complicated) constructs. Thus there is no problem is writing

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases} .$$

But better avoid

$$f(x) = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases} + 1$$

(whereas

$$f(x) = 1 + \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

is fine), and don’t omit the final ‘}’ in

$$f(x) = \left\{ \begin{array}{ll} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{array} \right\} + \left\{ \begin{array}{ll} x^2 & \text{if } x \geq 3 \\ -2 & \text{if } x < 3 \end{array} \right\} .$$

20. When formula is too long even for a display, break it within the display into several lines.

- Subsequent lines, after the first, should be indented properly.
- Break should occur after relation symbol or binary operation symbol, not before.

- (as in ordinary text) break should not leave an opening delimiter at the end of first line, or have the second line start with a closing delimiter or a punctuation mark
 - A unary operation symbol ($-x$, $n!$), sub- or superscript (incl. exponent) should not be separated from its argument (or base).
 - Similarly, in a function value " $f(x_1, \dots, x_n)$ ", the function name should not be separated at least from its first argument (together with its following comma and the preceding opening parenthesis)
 - break should not occur right after (but rather before) $\sum$, $\prod$ and $\int$.
 - if break occurs at a binary relation or operation symbol, it is helpful to display the symbol twice, at the end of the first line, and at the beginning of the second. In case of the product it is more helpful to use $\times$ instead of the centered dot " $\cdot$ ", or instead of omitting the dot.
 - try to place break so that the number of nested delimiter groups (i.e. opening delimiters which find their closing counterpart on the next line) is minimal at the break
 - a line break cannot occur within a fraction or a radical expression; in this case rewrite them (see rule 7 in §4.5)
21. Matching delimiters (in mathspeak as in ordinary text) should have equal size and shape.
- exception (for shape): half-open intervals " $[0, \infty)$ "
 - In general, matching delimiters should have equal size, which should be not much smaller than the expressions they enclose. In particular,
 - for nested delimiters in math formulas, outer delimiters should be not smaller than inner ones.
 - Avoid too many (even if it is allowed to have some) nested delimiters of equal size and shape.
 - For equal size delimiters put parentheses within brackets within braces (exception: delimiters with special meaning, like for intervals, Lie brackets, matrices).
 - Equal size and shape rule, as well as adjusting the delimiters size according to the entire enclosed expression, holds (in my opinion, at least) also if you break the delimited expression over a line (which is allowable). Thus I consider the left writing below appropriate, not the

right:

$$a = \Delta \cdot \left[(b-2)(b+3) + \frac{1}{c} + \frac{1}{(d+1)(d-3)} \right] \qquad a = \Delta \cdot \left[(b-2)(b+3) + \frac{1}{c} + \frac{1}{(d+1)(d-3)} \right]$$

In such situation try to balance the vertical width of the parts of the delimited text on both lines so as to avoid unnaturally large looking delimiters. For example, in

$$a = \Delta \cdot \left[\frac{1}{c} + (b-2)(b+3) + \frac{1}{(d+1)(d-3)} \right] \qquad a = \Delta \cdot \left[(b-2)(b+3) + \frac{1}{c} + \frac{1}{(d+1)(d-3)} \right]$$

on the left one uses equally high expressions to match both delimiter's size, while on the right the opening bracket remains too large. Instead of rearranging terms (or if this is not possible), if necessary, one can move a term of large height into a separate equation and introduce a symbol for it.

22. Write notations connected with arguments in a forward manner. Do not confront the reader with unnaturally complicated expressions, which are only subsequently justified. For example,
 - Instead for obtaining $|f(x) - g(x)| < \varepsilon$ to assume that $|x| < \sqrt{\frac{\varepsilon}{3M+2}}$, it is better to assume $|x| < \varepsilon$ and to conclude $|f(x) - g(x)| < (3M+2)^2 \varepsilon^2$ (and to remark that this still goes to 0 when so does ε).
23. Put end-of-proof marks (usually “□”) or, when the proof has ended, say “QED” (from the Latin ”quod erat demonstrandum”, meaning ”which was to be shown”) or “The proof is now complete.”
24. Many well-known notions differ slightly in their definition in previous literature. You help the reader a lot if you carefully specify the conventions you choose. A carefully specified non-standard convention is still better than a not explained “standard” one, no matter how standard it is.
25. Avoid technical terms, in particular such you create yourself; strive for simple terminology that reflects the content of the concept.

4.6 Formulation of Results and Statements

1. Keep your language/format simple and consistent (even boring)
2. Keep distractions to a minimum; make the interesting content stand out, in particular
3. Avoid unnecessary assumptions, for example
 - “In any commutative semisimple ring with unit we have for any elements x and y the identity $(x + y)^2 = x^2 + 2xy + y^2$.” (what do semisimplicity and unit have to do here?)
4. Do not avoid a remark on trivial and “degenerate” cases; exclude them if need be. (Failure to do so may result in losing reader’s confidence.)
5. Insist on sufficiently explicit explanations that easily uncover such peculiar particular examples.
6. Use similar format in similar situations, for example
 - Bad: “Proposition 1: If A and B hold, then C and D hold.
Proposition 2: C’ and D’ hold, assuming that A’ and B’ are true.”
 - Good: “Proposition 1: If A and B hold, then C and D hold.
Proposition 2: If A’ and B’ hold, then C’ and D’ hold.”
7. Theorem/Proposition/Lemma/Corollary: Give clear and unambiguous statements of results, preferably short and easy to remember. These are what other people are reading your paper for; so you should ensure that these, at least, can be understood by the reader (Basic rule 1).
8. The statement of the Theorem/Proposition/Lemma/Corollary should not include comments (except for an occasional brief remark in parenthesis) or examples.
9. Similarly, phrases like “without loss of generality we may assume...” or “Moreover it follows from theorem 1 that...”, that allude to argumentation, are to be avoided in statements of results, i.e., *separate clearly the result from its proof*.
10. If several hypotheses are required and several conclusions obtained, is there a more general concept that subsumes them?

4.7 Explain Properly

1. Choose a target audience level of expertise/background (e.g., undergraduate, first year graduate, research specialist, etc), and aim your math to that level.
2. Explain potentially unfamiliar material in separate segment(s).
3. Consider the use of appendixes and preliminary sections (see “General structuring rules”).
4. Explain your goals and your results (see also “General structuring rules”).
5. Explain the status of any one of your claims. Indicate whether it is a conjecture, the previous theorem, or the next corollary. Say why a claim is obvious, promise the necessary proof or counterexample later in the paper, give a reference if it is in the literature or admit not knowing about its truth.
6. Avoid in research expositions (as opposed to textbooks on well-known material) things like “Exercise”. This obscures the status of the claim and – unjustifiably – attempts to transfer responsibility for its correctness to the reader.
7. let the reader have confidence in you (“honesty is the best policy” Halmos)
 - for example, is the only proof of a result is tedious calculation, say something like “Unfortunately, the only known proof of this result is the following tedious calculation”.
8. Claiming something is “obvious”, reread the paper some time later – is it still obvious? Do someone you talk to perceive it as obvious? Be sure it is at all true!
9. Repeating statements, explain the difference to the previous formulation (or the lack of any difference) and why the repetition is necessary.
10. Repeating (large parts of) proofs points to more general concepts.
 - extract repeating parts of proofs of theorem 1 and 2 into a lemma, which then apply in the proof of both theorems
 - avoid as far as possible phrases like “by the same method as in the proof of ...” or even “see the proof of ...”

4.8 Use Suggestive References

internal references:

1. If you refer to a numbered definition, section, lemma, etc., remember to capitalize the word “Definition”, “Section”, “Lemma” etc.; it is considered a proper name.
2. All these formal statements and texts are usually set off from the rest of the discussion by putting some extra white space before and after them. Assign sequential reference numbers to these headings, irrespective of their different natures, and use a hierarchical scheme whose first component is the section number. (Thus “Corollary 3-6” refers to the sixth prominent statement in Section 3, and indicates that the statement is a corollary.)
3. For labelled displayed expressions, the reference key (usually a number, but sometimes also some non-alphanumeric symbol) is normally put in parentheses. However, one better precedes a reference like “(1)” by some word qualifying its nature like “recurrence”, “assumption”, “inequality”, “term” (not all displayed expressions are equations!). This is sometimes not only needed to resolve ambiguity (or at the start of a sentence), but provides helpful information to the reader. Avoid, however, the ugly abbreviation “eq.” for “equation”.
4. Page flipping wastes the reader’s time and breaks concentration, in particular
5. Frequent numbered equation/proposition references tangle the reader; avoid in particular labeling many equations you don’t use
6. Avoid frequent/important references without a verbal description of the statement, e.g. Bellman’s equation, weak duality theorem, etc.
7. Avoid excessive reference to statements by their creator’s name (it’s not a monument to people’s work you need to build); do it only for very classical statements (like Cauchy-Schwarz inequality), and preferably refer by content (Implicit function theorem), helping the reader understand what is actually needed from the statement.
8. Dare to be repetitive (but don’t overdo it), in particular
9. Repeat (rather than repeatedly refer to) simple mathematical expressions
10. Remind the reader of unusual notation, and earlier analysis. Notation that hasn’t been used in several pages (or even paragraphs) should carry a reference or a reminder of the meaning.

11. Writing

$$\int_{a_1}^{a_2} |f(x)|^2 dx < \infty, \quad (1)$$

do not say later “the function g satisfies (1)” – g does not appear in (1), what to put it for? So say instead “the equation (1) remains true if we replace f by g ”, or give the property on f described by (1) a name, and say “ g belongs to $L^2(a_1, a_2)$ ”.

12. Avoid forward references to theorems and in particular to equations (that is, references to statements coming later in the text). This may be appropriate (for main statements) in a summary and outline of work to follow, but surely not in mathematical reasoning.

external references:

1. If you use or quote an important result of another person, you should give a(n external) reference. You acknowledge hereby that his work is relevant for what you do.
2. You should avoid giving the impression that such a result is obvious, a generally accepted fact, due to you, and so on. *Plagiarism is almost certainly the greatest sin in academia.*
3. Try to be fair to whom you credit what result. Choosing references is often to some extent a matter of subjective taste, and can also be turned into a political instrument, by which work of particular people can be put to the attention of the readers – or be hidden from them (“out of cite, out of mind”). Remember: *it is your main responsibility to inform your reader clearly and correctly, rather than to be concerned with some mathematician’s reputation* (be it your advisor, your best friend or whoever)!
4. The reference key, usually a numeral (or a sequence of the initial author’s letters, optionally followed by a numeral or a year number), is enclosed in square brackets. Within the brackets and after the reference key, one can place – optionally – specific page numbers, section numbers, or equation numbers, preceded by a comma (“[5, Section 3.7, p. 9]”).
5. The use of latter option is encouraged, that is, make references to other papers (and in particular books) exact - state page/equation/proposition/theorem number. This costs only *little* extra space, but potentially saves *a lot* of effort from the reader!
6. The reference list is placed at end of paper (possibly before appendices).

7. In reference list the items are ordered alphabetically by the (alphabetically first) author; for several papers by the same author preferably chronologically.
8. A reference list item typically looks like

[ref key] J. Author, *Paper title*, Journal Name (optional serial number) **volume(issue)** (year), firstpage–lastpage.
9. Explain the status of the reference in your current situation. For example, a good convention is “see []” (meaning “a good reference is []”), “*cf.* []” (“for related ideas we suggest the reader to compare []”), “... []” (“the source is []”).
10. Point out possible mistakes (including misprints) in results you refer to, if they can mislead the reader and he cannot identify them easily.
11. Try to avoid using “by the proof of” when the proof is in the reference and your statements can be rewritten to be directly quoted.
12. A “well-known” result that is not in the literature should be proved if needed (Basic rule 1; see also “explain properly” section).
13. Avoid citing bad papers. In case you know that a paper you like to refer to has serious flaws (incorrect mathematics, incompetent or sloppy writing), avoid citing it, unless it is absolutely necessary. If you really need some result from that paper which *you know* is flawed, it is *your* responsibility to fix the mistakes in its proof (to the extent you need it).
14. The question is then: *How do I know that a paper I use is (not) flawed?* This is a serious problem. Most readers do not have the expertise to verify all details of a paper, and a fundamental assumption mathematics (and science more generally) builds on is that one can rely on another author’s responsibility for the correctness (or correction) of what he/she publishes. This means that as an author you should never
 - knowingly write falsehoods into your papers (called *fraud*), and
 - when you become (or are made by others) aware of an error in your paper, even made unintentionally, refuse to correct that error (called *cover-up*).

Beside plagiarism, fraud and covering up are serious academic crimes!

Unfortunately, discovering others’ flaws and attempts to openly discuss them and urge correction (called *whistleblowing*), although acts of scientific integrity, can lead to serious *ethical conflicts*. In recent years notorious

(fraud) cases have surfaced throughout science, where even established scientists aware of flaws in their papers have covered them up, and battled (often ruining their career) less reputed whistleblowers discovering these errors.

Acknowledgement and credits:

1. It's extremely important to acknowledge also support for the writing of the paper that goes beyond the presentation of its mathematical content. Such support includes among others
 - your inspiration (conversations with colleagues, being pointed to problems and helpful results),
 - invitations to talks and conferences where you discussed the material,
 - typing and proofreading your paper,
 - mental and financial support.

Example:

“Herewith I acknowledge that the people I quoted in the beginning of my lecture put their material on the web, so that I often only needed to copy-paste it without much own rewriting.”

2. Acknowledgement and credits can be given in a paragraph right before reference list, or at end of the introduction.

4.9 Consider Examples and Counterexamples

1. Before introducing a general concept, a properly chosen example can illustrate the main points, and gain the reader's confidence. “Even a simple example will get three-quarters of an idea across.” (Ullman)
2. Examples should have some spark, i.e., aim at something the reader may have missed
3. Illustrate definitions/results with examples that clarify the boundaries of applicability
4. Sometimes even a proof by example is more enlightening than an awkward general exposition. However,
5. Take care that the examples (even if simple) are “representative”, that is, have all the features needed in your arguments, and lack special other features that are not (to be) used.

6. Use counterexamples to clarify the limitations of the analysis, and the need for the assumptions.
7. Conclude about problems that arise from the (non-)existence of (counter)examples.

4.10 Use Visualization when possible

1. “A picture is worth a thousand words.”
2. Use a figure to illustrate the main idea of a proof/argument with no constraint of mathematical formality
3. Prefer graphs over tables
4. A good graph should convey relevant and specific information to the reader. You should point out significant graphical features.
5. Graphs and diagrams need to be neatly drawn and clearly labeled (for example, for coordinate plane diagrams, indicate the scale on the axes).
6. Use substantial captions (reinforcing, not repeating the reference in the text).
7. Keep figures simple. Don’t put too much information into one illustration, because papers without white space tire readers. For the same reason,
8. use adequate borders. Smooth the transitions between your words and pictures.
9. Place the illustrations closely after – or only a bit before – their references in the text.

4.11 Mathematical Publishing (Defend your Style)

1. While writing papers depends mainly on yourself, publishing depends mainly on other people - editors, referees and editorial assistants; the following are thus mostly hints how to deal with such people. (They are based to large extent on my own – not at all very positive – experience.)
2. **Warning:** While in the East manners and discipline seem to still be a status quo, this is far less so in the West. *Generally, you must know that Western people (including referees and editors) can be incorrect and aggressive!* Naturally, the worst-behaving people are those who can most afford it, i.e., those with the most authority. Even with your utmost effort at good conduct, do not expect such from them!

3. The editor makes final decision on whether to publish your paper, but is often not competent enough to examine the paper closely.
4. For this task, he chooses a referee; a referee is a (thought-to-be) expert, who is supposed to evaluate the paper in detail according to the demands of the journal.
5. An ideal referee reads a paper into all details, understands them fully and gives a neutral but competent opinion. In practice such referee hardly exists.
6. Both editors and referees usually work for no money and perform their job beside all their other obligations (they are paid for).
7. A usual referee is too busy with his own work; do not expect him to be very interested in your paper or to read fast.
8. A referee may indeed (although seldom) be extremely helpful. For example, beside his correcting tasks, he may point to improvements in your theorems – just expecting an impersonal credit, while he could have written the stronger result under his own name.
9. A referee (usually) knows the author, but not vice versa. The formal excuse for this practice is to prevent the author from harming the referee by getting indignant from his criticism. In reality, however, the referee often abuses his position to influence his opinion by his personal views about the author.
10. A general referee does not feel a duty to go through your paper and pin down all problems one by one. Behaving by the present rules of (non-existent) ethics, he could smash the paper with a lapidary comment, and then spread, without much consideration for your situation, degree of experience etc., bad rumors about your work. To avoid this, take serious care of your writing. *Do not fuel the referee's anger with your own sloppiness!*
11. Editors receive many more papers than they can publish; indeed they have to make a very stringent selection of material when they have a limited amount of space. Thus they welcome any reason to reject papers (unless when some of them handle papers written by their friends). Usually, after a negative referee's report hardly any editor will listen to author's objections, justifiable or not, and so authors must be prepared for rejections without any convincing arguments. The (in particular negative) *decisions on papers depend on much more than the papers themselves, and the author is always the scapegoat.*

12. Take referee's comments as seriously as you feel he has taken your paper. Neither ignore them nor follow them blindly. Be self-critical. Your paper may indeed have serious flaws. Therefore, do not always first blame others.
13. No matter how angry, even justifiably, you feel about a rejection of your paper, *try to avoid conflicts with editors and referees*. You have more to lose from such conflicts than they.
14. *Have confidence in your work! Do not sell it under value!* Choose carefully what journal to submit to. Sometimes recommendations of journals to submit are given by editors and referees who reject your paper. You must be aware that these people are primarily concerned that their journal looks the way they like, and not that you get your effort properly rewarded. My own attitude was, from a very early stage, in general not to be serious about following such recommendations. I must say that I am satisfied with the outcome.
15. Once (if) a paper is accepted, an editorial assistant (typesetter) prepares it for publication. He is a paid full-time worker concerned among others with checking all of grammatical – but not mathematical – exposition.
16. An editorial assistant has his own ideas about style. He is not a mathematics expert, and may consider corrections appropriate that distort the mathematics in your paper. Thus the application of his ideas about style on your paper may have negative consequences – *defend your style!*
 - Example: “ p or q holds depending on whether x is negative or positive” vs “ p or q holds depending on whether x is positive or negative”
17. Therefore, on the opposite hand, do not overload a typesetter to catch your mathematical sloppiness (Basic rule 2).
18. Nowadays, there is also the editorial “assistant” called “computer”. It is increasingly used in the production process, and sometimes particularly good at unpredictably altering the appearance of your paper. Try to keep your typesetting simple. The fancier you make it, the more error-prone is its computerised processing (Basic rule 0).
19. The editorial assistant sends you briefly prior to publication a final version of your paper for correcting (“galley proof”). Read it carefully to eliminate all mistakes that you have overlooked, or that were introduced during the production process. Send your corrections to the editorial assistant.
20. My experience is that often about 10% of these corrections are forgotten or improperly performed. To be absolutely sure ask the editorial assistant for a second galley proof.

21. After the paper is printed, (usually) reprints are delivered. Remind the editorial office if you have not received them in time. (Most journals accept such requests until half a year after publication.)