

Index

- A**
- $\bar{A}$ (closure), 95
 - Absolute retract, 223
 - vs. universal extension property, 223, 224
 - Accumulation point of a net, 188
 - Action of a group on a space, 199
 - Adjoining a 2-cell, 441
 - effect on fundamental group, 439
 - Adjunction space, 224
 - Affinely independent, 309
 - Alexander horned sphere, 393
 - Algebraic number, 51
 - $\hat{\alpha}$, 331
 - independence of path, 335
 - is isomorphism, 332
 - Antipodal map, 372
 - Antipode, 356
 - Antipode-preserving, 356
- B**
- Baire category theorem, 296
 - special case, 178
 - Baire space, 295
 - compact Hausdorff space, 296
 - complete metric space, 296
 - fine topology on $\mathcal{C}(X, Y)$, 300
 - locally compact Hausdorff space, 299
- Arc**, 308, 379
- does not separate S^2 , 389
- Archimedean ordering, 33
- Arzela's theorem, 280, 293
- Ascoli's theorem, 278, 290
- Axiom of choice, 59
- finite, 61
 - vs. nonemptiness of product, 118
 - vs. well-ordering theorem, 73

- Baire space (cont.)
 - open subspace of Baire space, 297
- $\mathbb{R}^J$ in box, product, uniform topologies, 300
- Ball, unit, 135, 156 (*see also* B^n)
- Barber of Seville paradox, 47
- Base point, 331
- Base point choice:
 - effect on h_* , 335
 - effect on π_1 , 332
- Basis:
 - for a free abelian group, 411
 - for a topology, 78, 80
- Bd A , 102
- $\beta(X)$ (*see* Stone-Čech compactification)
- Betti number, 424
- Bicompatness, 178
- Bijection function, 18
- Binary operation, 30
- Bing metrization theorem, 252
- Bisection theorem, 358, 359
- B^n , 156
 - compactness, 174
 - fundamental group, 331
 - path connectedness, 156
- Borsuk lemma, 382, 385
- Borsuk-Ulam theorem, 358, 359
- Boundary:
 - of a set, 102
 - of a surface with boundary, 476
- Bounded above, 27
- Bounded below, 27
- Bounded function, 267
- Bounded metric, 121, 129
- Bounded set, 121
- Box topology, 114
 - basis for, 115, 116
- Hausdorff condition, 116
 - subspace, 116
- vs.* fine topology, 290
- vs.* product topology, 115
- vs.* uniform topology, 124, 289, 290
- Brouwer fixed-point theorem, 351, 353
- B^2 , 135 (*see also* B^n)
- $B(x, \epsilon)$, 119
- C**
- Cantor set, 178
- Cardinality:
 - comparability, 68
 - of a finite set, 39, 42
 - greater, 62
 - same, 51
- Cartesian product:
 - countably infinite, 38
 - finite, 13, 37
 - general, 113
- Cauchy integral formula, 405
- Cauchy sequence, 264
- $\mathcal{C}(E, p, B)$, 487 (*see also* Group of covering transformations)
- Choice axiom (*see* Axiom of choice)
- Choice function, 59
- Circle, unit (*see* S^1)
- Classification:
 - of covering spaces, 482
 - of covering transformations, 488
 - of surfaces, 469
- Clockwise loop, 405
- Closed edge path, 566
- Closed graph theorem, 171
- Closed interval, 84
- Closed map, 137
- Closed ray, 86
- Closed refinement, 245
- Closed topologist's sine curve, 381
- separates S^2 , 381, 393
- Closed set, 93
 - in subspace, 94
 - vs.* limit points, 98
- Closure, 95
 - in a cartesian product, 101, 116
 - of a connected subspace, 150
 - in a subspace, 95
 - of a union, 101, 245
 - via* basis elements, 96
 - via* nets, 187
- Closure (cont.)
 - via* sequences, 130, 190
 - via* limit points, 97
- Coarser topology, 77
- Cofinal, 187
- Coherent topology, 224, 435
- Collection, 12
- Commutator, 422
- Commutator subgroup, 422
- Compact, 164 (*see also* Compact Hausdorff space, Compactness)
- Compact convergence topology 283
 - convergent sequences in, 283
 - independence of metric, 286
 - vs.* compact-open topology, 285
 - vs.* pointwise convergence topology, 285
- vs.* uniform topology, 285
- Compact Hausdorff space:
 - is Baire space, 296
 - components equal
 - quasicomponents, 236
 - metrizability, 218
 - normality, 202
 - paracompactness, 252
- Compactification, 185, 237
 - induced by an imbedding, 238
- one-point, 185
- of $(0, 1)$, 238
- Compactly generated space, 283
- Compactness, 164 (*see also* Compact Hausdorff space)
 - of closed intervals in $\mathbb{R}$, 173
 - closed set criterion, 169
 - of continuous image, 166
 - of countable products, 280
 - in $\mathcal{C}(X, \mathbb{R}^n)$, 278, 279
 - in $\mathcal{C}(X, Y)$, 290, 293
 - in finite complement topology, 166
 - of finite products, 167
 - in Hausdorff metric, 281
 - and least upper bound property, 172
 - in order topology, 172
 - and perfect maps, 172
- of product space, 234, 236
- in $\mathbb{R}$ and $\mathbb{R}^n$, 173
- of subspace, 164
- via* nets, 188
- vs.* completeness, 276
- vs.* limit point compactness, 179
- vs.* sequential compactness, 179
- Compact-open topology, 285
 - continuity of evaluation map, 286
 - vs.* compact convergence topology, 285
- Comparability:
 - of cardinalities, 68
 - of topologies, 77
- Comparison test for infinite series, 135
- Complement, 10
- Complete graph, 394
 - on five vertices, 308, 397
- Completely normal space, 205
- Completely regular space, 211 (*see also* Complete regularity)
- Complete metric space, 264 (*see also* Completeness)
- Completeness:
 - and Baire condition, 296
 - of $\mathcal{B}(X, Y)$ in uniform metric, 267
 - of closed subspace, 264
 - of $\mathcal{C}(X, Y)$ in uniform metric, 267
 - of $\mathcal{C}(X, Y)$ in sup metric, 268
 - of ℓ^2 , 271
 - of $\mathbb{R}^n$, 265
 - of $\mathbb{R}^\omega$, 265
 - of $\mathbb{R}^J$ in uniform metric, 267
 - of Y^X in compact-open topology, 289
 - of Y^J in uniform metric, 267
 - vs.* compactness, 276
- Complete regularity, 211
 - of locally compact Hausdorff space, 213
 - of product space, 211
 - of $\mathbb{R}^J$ in box topology, 213

- Complete regularity (cont.)
 of $\mathbb{R}_t^2$, 212
 of $S_\Omega \times \bar{S}_\Omega$, 212
 of subspace, 211
 of topological group, 213
 vs. normality, 211, 212
 vs. regularity, 214
 Complete set of relations, 425
 Completion, 269
 uniqueness, 271
 Component, 159
 of $\mathbb{R}^\omega$ in box topology, 162
 of $\mathbb{R}^\omega$ in uniform topology, 162
 vs. path component, 161
 vs. quasicomponent, 163, 236
 Composite:
 of functions, 17
 of continuous functions, 107
 of covering maps, 341, 485, 487
 of quotient maps, 141
 Conclusion, 7
 Cone, 499
 Conjugacy class, 481
 Conjugate elements, 419
 Conjugate subgroups, 481
 Connected component, 159
 Connectedness, 148
 in box topology, 151
 of closure, 150
 of continuous image, 150
 in finite complement topology, 152
 of finite products, 150
 in a linear continuum, 153
 of long line, 159
 of ordered square, 156
 of a product space, 152
 of $\mathbb{R}_K$, 177
 of $\mathbb{R}^\omega$, 151
 of subspace, 148
 of topologist's sine curve, 156
 vs. path connectedness, 156
 Connected sum:
 of projective planes, 452
 of tori, 451
 Connected space, 148 (*see also* Connectedness)
 Constant path, 327
 Contains, 4
 Continuity:
 of algebraic operations in $\mathbb{R}$, 131, 135
 basis criterion, 103
 and change of range, 108
 and closedness of graph, 171
 closed set criterion, 104
 closure criterion, 104
 of composites, 107
 of constant function, 107
 ϵ - δ formulation, 129
 of inclusion, 107
 local formulation, 108
 of maps from quotient spaces, 142
 of maps into products, 110, 117
 of metric, 126
 of $\min\{f, g\}$, 112
 at a point, 104
 of products of maps, 112
 of restriction, 108
 subbasis criterion, 103
 of uniform limit, 132
 in variables separately, 112
 via nets, 188
 via sequences, 130, 190
 Continuous function, 102 (*see also* Continuity)
 Continuous image:
 of a compact space, 166
 of a connected space, 150
 of a Lindelöf space, 194
 of a space with a countable dense subset, 194
 Continuum hypothesis 62, 205
 Contractible space, 330
 homotopy type, 366
 Contraction, 182, 270
 and fixed points, 182
 vs. shrinking map, 182
 Countably locally finite, 245
 Counterclockwise loop, 404
 Counterimage, 19
 Covering, 164
 of subspace, 164
 Covering dimension, 305
 Covering map, 336
 composite, 341, 485, 487
 is local homeomorphism, 338
 is open, 336
 products of, 339
 restrictions, 338, 484
 Covering space, 336
 classification, 482
 equivalence, 478
 existence, 495
 of figure eight, 340, 374, 375, 492, 493
 k -fold, 341
 of linear graph, 505
 of P^2 , 372
 regular, 489
 of $\mathbb{R}^2 - 0$, 340
 of S^1 , 337, 338, 482
 topological properties, 341, 500
 of torus, 339, 483
 universal, 484
 Covering transformation, 487
 Cube in $\mathbb{R}^n$, 314
 Curve, 225
 simple closed, 379
 Curved triangle, 471
 Cutting a region apart, 458
 CW complex, 445
 Cyclic group, 346
D
 $\bar{d}$, 121
 Decomposition space, 139
 Deformation retract, 361
 fundamental group of, 361
 Deformation retraction, 361
 vs. homotopy equivalence, 365, 366
 Degree of a map, 367
 Contrapositive, 8
 Convergent net, 187
 Convergent sequence, 98
 in compact convergence topology, 283
 in Hausdorff space, 99
 in point-open topology, 282
 in a product space, 118, 265
 Converges uniformly, 131
 Converse, 9
 Convex set:
 in an ordered set, 90, 153
 in $\mathbb{R}^n$, 325
 Coordinate:
 of J -tuple, 113
 of m -tuple, 37
 of ω -tuple, 38
 Coordinate function, 110
 Coset, 146, 330
 Countable basis, 190 (*see also* Second-countability)
 Countable basis at a point, 130, 190
 (*see also* First-countability)
 Countable compactness, 181
 Countable dense subset, 192
 effect of continuous function, 194
 in $\mathbb{R}^I$, 195
 in $\mathbb{R}^J$, 195
 in $\mathbb{R}_t$, 192
 in subspace, 194
 Countable intersection property, 235
 Countable set, 45 (*see also* Countability)
 Countability, 45
 of algebraic numbers, 51
 of countable unions, 48
 of finite products, 49
 of rationals, 48
 of subsets, 48
 via injective and surjective maps, 45
 of $\mathbb{Z}$, 44
 of $\mathbb{Z}_+ \times \mathbb{Z}_+$, 45, 48
 Countably infinite, 44
 Countably locally discrete, 252

- DeMorgan's laws, 11
- Dense subset, 191
- Diagonal, 101
- Diameter of a set, 121
- Dictionary order, 26
- Difference of two sets, 10
- Dimension, topological, 305 (*see also* Topological dimension)
- Directed set, 187, 188
- Direct sum, 408, 409
 - existence, 409
 - extension condition, 408, 410
 - uniqueness, 410
- Discrete topology, 77
 - metric for, 120
- Disjoint sets, 6
- Distance, 119
- Distance from x to A , 175
- Distributive laws for $\cup$ and $\cap$, 11
- Domain, 16
- Double torus, 374, 452
- Fundamental group, 374
- Doubly punctured plane, 362
- Dunce cap, 443
- Fundamental group, 444
 - $d(x, A)$, 175
- E**
- Edge:
 - of curved triangle, 471
 - of a linear graph, 308, 394, 502
 - of a polygonal region, 447
- Edge path, 506
 - reduced, 507
- Element of a set, 4
- Elementary divisors, 424
- Elementary operations on schemes, 460
- Empty interior, 295
- Empty set, 6
- End points of arc, 308, 378
- Epimorphism, 330
- ϵ -ball, 119
- ϵ -neighborhood of a set, 177
- Equality symbol, 4
- Equicontinuity, 276
 - vs.* compactness, 278, 279
 - vs.* total boundedness, 277
- Equivalence class, 22
- Equivalence of compactifications, 237
- Equivalence of covering maps, 478
 - existence, 480, 482
- Equivalence of labelling schemes, 461
- Equivalence relation, 22
- Euclidean metric, 122, 128
- Euclidean space, 38
- Euler number, 506, 514, 515
- Evaluation map, 271, 286
- Evenly covered, 336
- Eventually zero, 51
- e_x (constant path), 327
- Expansion lemma, 260
- Extension condition:
 - for direct sums, 408, 410
 - for free abelian groups, 411
 - for free groups, 421
 - for free products, 414, 418, 419
- External direct sum, 409
- External free product, 415
- Extreme value theorem, 174
- F**
- $[f]$, 324
- Family of sets, 36
- $f * g$, 326
- Fibonacci numbers, 56
- Field, 31
- Figure-eight space, 340, 362
 - covering space, 340, 374, 375, 492, 493
- Fundamental group, 373
- Final point:
 - of oriented line segment, 447, 506
 - of path, 323
- Finer topology, 77
 - basis criterion, 81
- Fine topology, 289
 - is Baire, 300
- Free generators for a group, 421
- Free group, 421
 - extension condition, 421
 - on a set, 422
 - subgroup is free, 514
- Free homotopy of loops, 403
- Free product, 413
 - existence, 415
 - extension condition, 414, 418, 419
 - external, 415
 - uniqueness, 418
- Frobenius theorem, 351
- F_σ set, 252
- Function, 16
- Functor, 242
- Functorial properties of h_* , 334
- Fundamental group, 331
 - of dunce cap, 444
 - of deformation retract, 361
 - of double torus, 374, 452
 - of figure eight, 362, 373, 434
 - of infinite earring, 500
 - of linear graph, 511
 - of m -fold projective plane, 453
 - of n -fold torus, 452
 - of a product, 371
 - of P^2 , 373
 - of $\mathbb{R}^n - 0$, 360
 - of S^1 , 345
 - of S^n , 369
 - of theta space, 362, 432
 - of torus, 371, 442
 - of wedge of circles, 434, 436
 - of wedge of spaces, 438
 - when abelian, 335
 - when countable, 499, 500
 - when finitely generated, 500
 - when uncountable, 500
- Fundamental theorem of algebra, 354
- G**
- G_δ set, 194, 249
- Generalized continuum hypothesis, 62
- General lifting lemma, 478
- Finite axiom of choice, 61
- Finite complement topology, 77
 - compactness, 166
 - connectedness, 152
- Finite dimensional, 305
- Finite intersection property, 169
- Finitely generated group, 421
- Finitely presented group, 425
 - as fundamental group, 445
- Finiteness:
 - of cartesian products, 43
 - of subsets, 43
 - of unions, 43
 - vs.* injective and surjective maps, 43
- Finite presentation, 425
- Finite set, 39
- First category set, 295
- First coordinate, 13
- First-countability, 131, 190
 - implies compactly generated, 283
 - of metric space, 131
 - of product, 191
 - of subspace, 191
 - of $\mathbb{R}_\ell$, 192
- First-countable space
 - (*see* First-countability)
- First homology group, 455
 - of m -fold projective plane, 456
 - of n -fold torus, 456
- First homotopy group (*see* Fundamental group)
- Fixed point, 158, 182
- Fixed-point-free action, 493
- Fixed point theorem:
 - for B^n , 353
 - for B^2 , 351
 - for a contraction, 182, 270
 - for retract of B^2 , 353
 - for a shrinking map, 182
 - for $[0, 1]$, 158
- Free abelian group, 411
 - extension condition, 411
 - rank, 411
 - subgroup is free abelian, 412

- General linear group, 146
- General nonseparation theorem, 390
- General position, 308, 310
- General separation theorem, 380, 392
- Generated:
 - by elements, 411, 421.
 - by subgroups, 407, 412
- Generator of cyclic group, 346
- Geometrically independent, 309
- $[G, G]$, 422
- G/H , 146, 331
- regularity, 146
- as topological group, 146
- Graph of a function, 171
- Greater cardinality, 62
- Greatest lower bound, 27
- property, 27
- Group of covering transformations, 487
- Groupoid properties 326
- H**
- h_* , 333
- dependence on base point, 335
- functorial properties, 334
- Hahn-Mazurkiewicz theorem, 275
- Half-open interval, 84
- Ham sandwich theorem, 359
- Hausdorff condition, 98
- for box topology, 116
- and closedness of diagonal, 100
- and convergent sequences, 99
- for manifold, 227
- for metric space, 129
- for orbit space, 199
- for order topology, 100
- and perfect maps, 199
- for product space, 100, 116, 196
- for quotient space, 142
- for subspace, 100, 196
- for topological group, 146
- and uniqueness of extensions, 112, 240
- vs. regularity, 195, 197
- vs. T_1 axiom, 99
- Hausdorff maximum principle, 69
- Hausdorff metric, 281
- Hausdorff space, 98 (*see also* Hausdorff condition)
- Have the same cardinality, 51
- Hilbert cube, 128
- Homeomorphism, 105
- vs. continuous bijective map, 106, 167
- Homogeneous space, 146
- Homology group, 455
- Homomorphism, 330
- induced by a map, 333 (*see also* h_*)
- induced by a path, 331 (*see also* $\hat{\alpha}$)
- Homotopic maps, 323
- Homotopy, 323
- effect on h_* , 360, 363, 364
- as path in function space, 288
- straight-line, 325
- Homotopy equivalence, 363
- induces isomorphism of π_1 , 364
- vs. deformation retraction, 365, 366
- Homotopy extension lemma, 381
- Homotopy inverse, 363
- Homotopy type, 363
- of contractible space, 366
- $H_1(X)$ (*see* First homology group)
- Hypothesis, 7
- I**
- $I^2_\circ$ (*see* Ordered square)
- Identification space, 139
- Identity function, 21
- "If ... then," meaning of, 7
- Image, 16, 19
- Imbedding, 105
- isometric, 133
- Imbedding theorem:
 - for a compact manifold, 226, 314
 - for a completely regular space, 217
 - for a linear graph, 308
 - for a manifold, 316
 - for a space of dimension m , 311
- Immediate predecessor, 25
- Immediate successor, 25
- Inclusion, 4
- Indexed family of sets, 36
- Indexing function, 36
- Index set, 36
- Index of a subgroup, 514
- Indiscrete topology, 77
- Induction principle, 32
- strong, 33
- transfinite, 67
- Inductive definition, 47 (*see also* Recursive definition)
- Inductive dimension, 315
- Inductive set, 32, 67
- Inf A , 27
- Infimum, 27
- Infinite broom, 162
- Infinite earring, 436
- fundamental group, 500
- no universal covering, 487
- Infinite sequence, 38
- Infinite series, 135
- Infinite set, 44
- via injective and bijective functions, 57
- Initial point:
 - of an oriented line segment, 447, 506
 - of a path, 323
- Injective function, 18
- Int A , 95
- Integers, 32
- Interior point:
 - of an arc, 379
 - of a set, 95
- Intermediate-value theorem, 147, 154
- Intersection, 6, 12, 36
- Interval, 25, 84
- Intervals in $\mathbb{R}$:
 - compactness, 173
 - connectedness, 154
 - topological dimension, 305
- Invariance of domain, 383, 385
- Inverse function, 18
- Inverse image, 19
- Isolated point, 176
- Isometric imbedding, 133
- in complete metric space, 269, 271
- Isometry, 181
- Isomorphism, 105, 330
- J**
- Jordan curve theorem, 390
- Jordan separation theorem, 379
- J -tuple, 113
- K**
- Kernel of homomorphism, 330
- k -fold covering, 341
- Klein bottle, 454
- k -plane, 310
- K -topology on $\mathbb{R}$, 82 (*see also* $\mathbb{R}_K$)
- Kuratowski 14-set problem, 102
- Kuratowski lemma, 72
- L**
- Labelling, 447
- Labelling scheme, 449
- (*see also* Scheme)
- Labels, 447
- Larger topology, 77
- Largest element, 27
- Least normal subgroup, 419
- generators, 420
- Lebesgue number, 175
- Lebesgue number lemma, 175
- Least upper bound, 27
- Least upper bound property, 27
- and compactness, 172, 177
- and local compactness, 183
- for $\mathbb{R}$, 31
- for well-ordered sets, 66
- vs. greatest lower bound property, 29
- Left coset, 146, 330
- Left inverse, 21
- Length of a word, 412
- Lens space, 494

Lifting, 342
 Lifting correspondence, 345
 Lifting lemma:
 general, 478
 for path homotopies, 343
 for paths, 342
 Limit point, 97
 vs. T_1 axiom, 99
 Limit point compactness, 178
 vs. compactness, 179
 vs. countable compactness, 181
 Limit of a sequence, 100
 Lindelöf condition, 192 (*see also* Regular Lindelöf space)
 for closed subspace, 194
 effect of continuous function, 194
 for products, 193
 for $\mathbb{R}$, 192
 for $\mathbb{R}_\ell^2$, 193
 for subspace, 193
 Linear continuum, 31, 153
 compact subspaces, 172
 connected subspaces, 153
 long line, 158
 normality, 206
 ordered square, 155
 Linear graph, 308, 394, 502
 covering space of, 505
 fundamental group, 511
 imbedding in $\mathbb{R}^3$, 308
 local path connectedness, 504
 local simple connectedness, 504
 semilocal simple
 connectedness, 504
 topological dimension, 308
 Linear order, 24
 Line with two origins, 227
 Little ell-two topology, 128
 Local compactness, 182
 implies compactly generated, 283
 and least upper bound property, 183
 for orbit space, 199
 and perfect maps, 199
 of products, 186

of $\mathbb{R}$ and $\mathbb{R}^n$ and $\mathbb{R}^\omega$, 182
 of subspace, 185
 Local connectedness, 161
 of quotient space, 163
 vs. weak local connectedness, 162
 Local homeomorphism, 338
 Locally compact Hausdorff space:
 Baire condition, 299
 complete regularity, 213
 regularity, 205
 Locally discrete, 254
 Locally euclidean, 316
 Locally finite collection, 244
 Locally finite family, 112
 vs. locally finite collection, 245
 Local metrizable, 218, 261
 Local path connectedness, 161
 Local simple connectedness, 495
 vs. simple connectedness, 499
 Logical equivalence, 8
 Logical quantifiers, 9
 Long line, 158, 317
 connectedness, 159
 path connectedness, 159
 Loop, 331
 Lower bound, 27
 Lower limit topology, 82 (*see also* $\mathbb{R}_\ell$)
 ℓ^2 -topology, 128

M

Manifold 225, 316
 imbedding in $\mathbb{R}^n$, 226, 314, 316
 metrizable, 227
 necessity of Hausdorff condition, 227
 regularity, 227
 topological dimension, 314, 316
 Mapping, 16
 Maximal element, 70
 Maximal tree, 509
 Maximum principle, 69
 vs. well-ordering theorem, 73
 vs. Zorn's lemma, 70, 72

Maximum value theorem:
 of calculus, 147
 general, 174
 Metric, 119
 bounded, 129
 for discrete topology, 120
 for $\mathbb{R}$, 120
 for $\mathbb{R}^n$, 122
 for $\mathbb{R}^\omega$, 125
 Metrically equivalent, 270
 Metric space, 120
 Hausdorff condition, 129
 normality, 202
 paracompactness, 257
 subspace, 129
 Metric topology, 119
 Metrizable space, 120
 Metrizability:
 of compact Hausdorff space, 218
 of manifolds, 227
 of ordered square, 194
 of products, 133, 134
 of regular Lindelöf space, 218
 of regular second-countable space, 215
 of $\mathbb{R}^J$, 133
 of $\mathbb{R}_\ell$, 194
 of $\mathbb{R}^n$, 123
 of $\mathbb{R}^\omega$, 125, 132
 of S_Ω and $\tilde{S}_\Omega$, 181
 of Stone-Čech compactification, 242
 m -fold projective plane, 452
 first homology group, 456
 fundamental group, 453
 Minimal uncountable well-ordered set, 66 (*see also* S_Ω)
 Möbius band, 450
 Monomorphism, 330
 m -tuple, 37

N

Nagata-Smirnov metrization theorem, 250
 Negation, 9

Neighborhood, 96
 Nested sequence of sets, 170
 Net, 187
 $n(f, a)$ (*see* Winding number)
 n -fold torus, 451
 first homology group, 456
 fundamental group, 452
 Nonseparation theorem:
 arc in S^2 , 389
 general, 390
 topologist's sine curve in S^2 , 393
 No-retraction theorem, 348
 Norm, 122
 Normality, 195
 of adjunction space, 224
 of closed subspace, 205
 of coherent topology, 224
 of compact Hausdorff space, 202
 of linear continuum, 206
 of linear graph, 502
 of metric space, 202
 of orbit space, 199
 of product, 198, 203
 of paracompact Hausdorff space, 253
 of quotient space, 199, 443
 of regular Lindelöf space, 205
 of regular second-countable space, 200
 of $\mathbb{R}_\ell$, 198
 of $\mathbb{R}^J$, 203
 of subspace, 203
 of topological group, 207
 vs. complete regularity, 211, 212
 vs. regularity, 195, 198, 203
 of well-ordered set, 202
 Normalizer, 487
 Normal space, 195 (*see also* Normality)
 Normal subgroup, 330
 Nowhere-differentiable function, 300
 Nulhomotopy lemma, 377
 Nulhomotopic map, 323
 induces trivial homomorphism, 364

O

- ω -tuple, 38
 - One-point compactification, 185
 - uniqueness, 183
 - One-to-one correspondence, 18
 - "Onto" function, 18
 - Open covering, 164
 - Open interval, 25, 84
 - Open map, 92, 137
 - Open ray, 86
 - Open refinement, 245
 - Open set, 76
 - relative to subspace, 89
 - Operation, binary, 30
 - Operation on schemes, 460
 - Orbit, 490
 - Orbit space, 199, 490
 - Order of a covering, 305
 - Ordered field, 31
 - Ordered pair, 13
 - Ordered square, 90
 - connectedness, 156
 - is linear continuum, 155
 - metrizable, 194
 - path connectedness, 156
 - Order of a group, 346
 - Order of a group element, 412
 - Order relation, 24
 - Order topology, 84
 - compact subspaces, 172
 - normality, 202, 206
 - Hausdorff condition, 100
 - subbasis, 86
 - vs. subspace topology, 91
 - Order type, 25
 - Oriented edge of a graph, 506
 - Oriented line segment, 447
 - "Or," meaning of, 5
- P**
- $\mathcal{P}(A)$, 12
 - Paracompactness, 253
 - of compact Hausdorff space, 252
 - of closed subspace, 254

- $\pi_1(X, x_0)$, 331 (*see also* Fundamental group)
 - Plane in $\mathbb{R}^N$, 310
 - Point-finite collection, 248
 - Point-finite family, 227
 - Point-open topology, 281
 - convergent sequences in, 282
 - equals product topology, 282
 - vs. compact convergence topology, 285
 - vs. compact-open topology, 285
 - Pointwise bounded, 278
 - Pointwise convergence topology, 281
 - (*see also* Point-open topology)
 - Polygonal region, 447
 - Positive integers, 32
 - Positive linear map:
 - of intervals in $\mathbb{R}$, 328
 - of oriented line segments, 447
 - Power set, 12
 - Precise refinement, 258
 - Preimage, 19
 - Presentation of a group, 425
 - Principle of induction, 32
 - transfinite, 67
 - Principle of recursive definition, 47, 54
 - general, 72
 - Product:
 - of continuous maps, 112
 - of covering maps, 339
 - of open maps, 141
 - of path-homotopy classes, 326
 - of paths, 326
 - of quotient maps, 141, 143, 145, 186, 289
 - Product space, 114 (*see also* Product topology)
 - fundamental group, 371
 - Product topology, 86, 114
 - basis, 86, 115, 116
 - closures in, 101, 116
 - compactness, 167, 234
 - complete regularity, 211
 - connectedness, 150, 152
- convergent sequences, 118, 265
 - first-countability, 191
 - Hausdorff condition, 100, 116, 196
 - Lindelöf condition, 193
 - local compactness, 186
 - metrizable, 133, 134
 - normality, 198, 203
 - paracompactness, 257
 - regularity, 196
 - second-countability, 191
 - subbasis, 88, 114
 - vs. box topology, 115
 - vs. point-open topology, 282
 - vs. quotient topology, 141, 143, 145, 186, 289
 - vs. subspace topology, 89, 116
 - vs. uniform topology, 124
 - Projection map, 87, 114
 - is open map, 92
 - Projective n -space, 373
 - Projective plane, 372 (*see also* P^2)
 - Projective-type scheme, 463
 - Proper inclusion, 4
 - Proper labelling scheme, 463
 - Proper subset, 4
 - Properly discontinuous, 490
 - Prufer manifold, 317
 - P^2 , 372
 - fundamental group, 373
 - is surface, 372
 - Punctured euclidean space, 156 (*see also* $\mathbb{R}^n - 0$)
 - Punctured plane, 325 (*see also* $\mathbb{R}^2 - 0$)
- Q**
- $\mathbb{Q}^\infty$, 195
 - Quantifiers, logical, 9
 - Quasicomponent, 163
 - vs. component, 163, 236
 - Quotient group, 331
 - Quotient map, 137
 - composites, 141
 - products, 141, 143, 145, 186, 289
 - restrictions, 137, 138, 140

Quotient space, 139 (*see also* Quotient topology)
 Quotient topology, 138
 and continuous functions, 142
 Hausdorff condition, 142, 199
 local compactness, 199
 local connectedness, 163
 normality, 199
 regularity, 199
 second-countability, 199
 T_1 condition, 141
 vs. product topology, 141, 143, 145, 186, 289

R

$\mathbb{R}$ (reals), 30
 algebraic properties, 30
 compact subspaces, 173
 connected subspaces, 154
 local compactness, 182
 K -topology, 82 (*see also* $\mathbb{R}_K$)
 lower limit topology, 82 (*see also* $\mathbb{R}_+$)

metric for, 120

order properties, 31

second-countability, 190

standard topology, 81

uncountability, 177

$\mathbb{R}_+$, 31

Range of a function, 16

Rank of a free abelian group, 411

Rational number, 32

Ray in ordered set, 85

Recursive definition, principle, 47, 54

 general principle, 72

Reduced edge path, 507

Reduced word, 413

Refinement, 245, 305

Regular covering space, 489

 is orbit space, 491

Regularity, 195

 of G/H , 146

 of locally compact Hausdorff space, 205

of manifold, 227
 of orbit space, 199
 and perfect maps, 199
 of products, 196
 of subspaces, 196
 of topological groups, 146
 vs. complete regularity, 214
 vs. Hausdorff condition, 195, 197
 vs. metrizable, 215
 vs. normality, 195, 198, 203
 Regular Lindelöf space:
 metrizable, 218
 normality, 205
 paracompactness, 257
 Regular space, 195 (*see also* Regularity)

Restriction:

 of a covering map, 338, 484

 of a function, 17

 of a quotient map, 137, 138, 140

 of a relation, 28

Retract, 223, 348

Retraction, 335, 348

 as quotient map, 144

Represented by a word, 412

Reverse of a path, 327

Relation, 21

Relation on a free group, 424

 complete set, 425

ρ , 122, 268 (*see also* sup metric)

$\bar{\rho}$, 124, 266 (*see also* uniform metric)

$\mathbb{R}'$, countable dense subset, 195

Right coset, 330

Right inverse, 21

$\mathbb{R}^\infty$, 118

 closure in $\mathbb{R}^\omega$, 118, 127

 paracompactness, 260

$\mathbb{R}'$ in box topology:

 is Baire, 300

 complete regularity, 213

$\mathbb{R}'$ in product topology:

 is Baire, 300

 countable dense subset, 195
 metrizable, 133

$\mathbb{R}'$ in product topology (cont.)
 normality, 203

 paracompactness, 257

$\mathbb{R}'$ in uniform topology, 124

 is Baire, 300

 completeness, 267

$\mathbb{R}_K$, 82

 connectedness, 178

 separation axioms, 197

 vs. standard topology, 82

$\mathbb{R}_\ell$, 82

 countability axioms, 192

 metrizable, 194

 normality, 198

 paracompactness, 257

 vs. standard topology, 82

$\mathbb{R}_\ell^2$, 193

 complete regularity, 212

 Lindelöf condition, 193

 paracompactness, 257

 separation axioms, 198

$\mathbb{R}^n$, 38

 basis, 116

 compact subspaces, 173

 local compactness, 182

 metrics for, 122, 123

 paracompactness, 253

 second-countability, 190

$\mathbb{R}^n - 0$, 156

 fundamental group, 360

 path connectedness, 156

$\mathbb{R}^\omega$, 38

$\mathbb{R}^\omega$ in box topology:

 components, 162

 connectedness, 151

 metrizable, 132

 normality, 205

 paracompactness, 205

$\mathbb{R}^\omega$ in product topology:

 completeness, 265

 connectedness, 151

 local compactness, 182

 metrizable, 125

 paracompactness, 257

second-countability, 190
 $\mathbb{R}^\omega$ in uniform topology:

 components, 162

 paracompactness, 257

 second-countability, 190

$\mathbb{R}^2$, standard topology, 87

$\mathbb{R}^2 - 0$, 325

 covering space, 340

 fundamental group, 360

 Rule of assignment, 15

 Russell's paradox, 62

S

S_α (section of well-ordered set), 66

Saturated set, 137

Scheme, 449

 projective type, 463

 proper, 463

 torus type, 463

Schroeder-Bernstein theorem, 52

Schoenflies theorem, 392

Second category set, 295

Second coordinate of ordered pair, 13

Second-countability, 190

 of compact metric space, 194

 of $\mathcal{C}(I, R)$, 194

 of orbit space, 199

 and perfect maps, 199

 of products, 191

 of $\mathbb{R}$ and $\mathbb{R}^n$ and $\mathbb{R}^\omega$, 190

 of $\mathbb{R}_\ell$, 192

 of $\mathbb{R}^\omega$ in uniform topology, 190

 of subspace, 191

 of topological group, 195

 vs. countable dense subset, 194

 vs. Lindelöf condition, 194

Second-countable space, 190 (*see also* Second-countability)

Section:

 of the positive integers, 32

 of a well-ordered set, 66

Seifert-van Kampen theorem 426

 classical version, 431

 special case, 369

Semilocally simply connected, 494
 Separable, 192 (*see also* Countable dense subset)
 Separates points from closed sets, 218
 Separates a space, 378
 into n components, 378
 Separation, 148
 by continuous functions, 211
 Separation theorem:
 closed topologist's sine curve in S^2 , 393
 general, 380, 392
 simple closed curve in S^2 , 379, 390
 theta space in S^2 , 395
 Sequences, 38
 and closure, 130, 190
 and continuity, 130, 190
 Sequence lemma, 130
 Sequential compactness, 179
 vs. compactness, 179
 Shrinking lemma, 227
 general, 258
 Shrinking map, 182
 and fixed points, 182
 vs. contraction, 182
 σ -compact, 289, 316
 σ -locally discrete, 252
 σ -locally finite, 245
 Simple closed curve, 379
 generates π_1 of $\mathbb{R}^2 - \{0\}$, 401
 separates S^2 , 379, 390
 winding number, 404, 406
 Simply connected, 333
 S^n , 369
 star-convex set, 334
 tree, 508
 vs. locally simply connected, 499
 Simple loop, 404
 Simple order, 24
 Slice:
 in covering space, 336
 in product space, 167
 Smaller topology, 77
 Smallest element of ordered set, 27

Smirnov metrization theorem, 261
 S^n (unit sphere), 156
 compactness, 174
 fundamental group, 369
 path connectedness, 156
 simple connectedness, 369
 S_Ω , 66
 compactification, 242
 countable subsets, 66, 74
 existence, 74
 metrizable, 181
 paracompactness, 260, 261
 uniqueness, 73
 $\bar{S}_\Omega$, 66
 metrizable, 181
 $S_\Omega \times \bar{S}_\Omega$, 203
 complete regularity, 212
 normality, 203
 paracompactness, 254
 S^1 , 106
 covering spaces, 337, 482
 fundamental group, 345
 Sphere, unit, 139, 156 (*see also* S^2 , S^n)
 Sorgenfrey plane, 193 (*see also* $\mathbb{R}^2$)
 Square metric, 122 (*see also* ρ)
 Standard bounded metric, 121
 Standard topology:
 on $\mathbb{R}$, 81
 on $\mathbb{R}^2$, 87
 Star-convex set, 334
 Stereographic projection, 369
 Stone-Čech compactification, 241
 existence, 239
 extension condition, 240
 metrizable, 242
 of S_Ω , 242
 of $\mathbb{Z}_+$, 242
 Straight-line homotopy, 325
 Strictly coarser topology, 77
 Strictly finer topology, 77
 Strict partial order, 68
 Strong continuity, 137
 Stronger topology, 78

Strong induction principle, 33
 S^2 , 139
 as quotient space, 136, 139
 fundamental group, 432
 separates S^2 , 395
 T_i axioms, 211, 213
 Tietze extension theorem, 219
 T_1 axiom, 99
 vs. Hausdorff condition, 99
 vs. limit points, 99
 for quotient space, 141
 Topological completeness, 270 (*see also* Complete metric space)
 Topological dimension, 305
 of closed subspace, 306
 of closed subspace of $\mathbb{R}^N$, 316
 of compact manifold, 314
 of compact subspace of $\mathbb{R}$, 305
 of compact subspace of $\mathbb{R}^M$, 313
 of compact subspace of $\mathbb{R}^2$, 306
 of linear graph, 308
 of manifold, 316
 of 1-manifold, 308
 of triangular region, 352
 of 2-manifold, 308, 352
 of a union, 307, 308
 of $[0, 1]$, 305
 Topological group, 145
 closedness of $A \cdot B$, 172, 188
 complete regularity, 213
 covering space of, 483
 Hausdorff condition, 146
 normality, 207
 π_1 is abelian, 335
 paracompactness, 261
 regularity, 146
 second-countability, 195
 Topological imbedding, 105
 Topological property, 105
 Topological space, 76
 Topologist's sine curve, 157
 components, 160

System of free generators, 421

T

Theta space, 362, 394
 fundamental group, 432
 separates S^2 , 395
 T_i axioms, 211, 213
 Tietze extension theorem, 219
 T_1 axiom, 99
 vs. Hausdorff condition, 99
 vs. limit points, 99
 for quotient space, 141
 Topological completeness, 270 (*see also* Complete metric space)
 Topological dimension, 305
 of closed subspace, 306
 of closed subspace of $\mathbb{R}^N$, 316
 of compact manifold, 314
 of compact subspace of $\mathbb{R}$, 305
 of compact subspace of $\mathbb{R}^M$, 313
 of compact subspace of $\mathbb{R}^2$, 306
 of linear graph, 308
 of manifold, 316
 of 1-manifold, 308
 of triangular region, 352
 of 2-manifold, 308, 352
 of a union, 307, 308
 of $[0, 1]$, 305
 Topological group, 145
 closedness of $A \cdot B$, 172, 188
 complete regularity, 213
 covering space of, 483
 Hausdorff condition, 146
 normality, 207
 π_1 is abelian, 335
 paracompactness, 261
 regularity, 146
 second-countability, 195
 Topological imbedding, 105
 Topological property, 105
 Topological space, 76
 Topologist's sine curve, 157
 components, 160

Topologist's sine curve (cont.)

connectedness, 156
 does not separate S^2 , 393
 path components, 160
 path connectedness, 157

Topology, 76

generated by a basis, 78, 80
 generated by a subbasis, 82
 Torsion subgroup, 412, 424

Torus, 339

covering space of, 339, 483
 equals doughnut surface, 339
 fundamental group, 371, 442
 as quotient space, 136, 140

Torus-type scheme, 463

Totally bounded, 275

vs. equicontinuity, 277

Totally disconnected, 152

Tower, 73

Transcendental number, 51

Transfinite induction, 67

Translation of $\mathbb{R}^N$, 310

Tree, 507

fundamental group, 508

maximal, 509

Triangle inequality, 119

Triangulable, 471

Triangulation, 471

Trivial homomorphism, 335

Trivial topology, 77

Tube, 167

Tube lemma, 168

generalized, 171

Tukey lemma, 72

2-cell, 441

2-manifold, 225

topological dimension, 308, 352

2-manifold with boundary, 476

2-sphere, 139 (*see also* S^2)

Tychonoff theorem, 234

for countable products, 280

for finite products, 167

via well-ordering theorem 236

Value of a function, 16

Vanish at infinity, 280

Vanish precisely on A , 213

Vector field, 350

Vertex:

of a curved triangle, 471

of a linear graph, 308, 394, 502

of a polygonal region, 447

W

Weaker topology, 78

Weak local connectedness, 162

vs. connectedness, 162

Wedge of circles, 434, 435

existence, 437

fundamental group, 434, 436

Wedge of spaces, 438

Weierstrass M -test, 135

Well-ordered set, 63

compact subspaces, 172

dictionary order, 64

finite, 64

normality, 202

subsets well-ordered, 63

uncountable, 66

$\mathbb{Z}_+$, 32

$\mathbb{Z}_+ \times \mathbb{Z}_+$, 63

Well-ordering theorem, 65

applied, 236, 246

and axiom of choice, 67, 73

and maximum principle, 70, 73

Winding number, 398, 403

as an integral, 405

of simple closed curve, 404, 406

Word, 412, 415

reduced, 413

X

X^j , 113

X^m , 38

X^ω , 38

$[X, Y]$, 330

Z

$\mathbb{Z}$, 32

$\mathbb{Z}_+$, 32

not finite, 42

well-ordered, 32

Zermelo, 65

Zorn's lemma, 70

applied, 72, 233, 236, 509

vs. maximum principle, 72

V

Vacuously true, 7