GS2004 **Linear Algebra** Fall 2019

Name (hangul): ___________ 학과/학년: ___________ Student #: ___________ Section: ___________

Quiz-11 (Review quiz) (12 problems for 20 minutes, 60 points, 100%=40 points) Dec 02

Multiple choice problems *(5 points for right answer, 0 points for no answer, -1 point for wrong answer)*

Problem 1. When is a field algebraically closed?

- ☐ when every non-constant polynomial has at least one zero
- ☐ when every polynomial of degree n has n distinct zeros
- ☐ when every polynomial of degree n has at most n zeros
- ☐ when there is no polynomial with a multiple zero which splits

Problem 2. Which of these claims is true for all $A \in M_{n\times n}(F)$?

- ☐ If A is traceless and upper-triangular, then A is not invertible.
- ☐ If A is scalar and traceless, then A is not invertible.
- ☐ If A is diagonalizable and invertible, then A is diagonal.
- ☐ If A is upper-triangular and symmetric, then A is diagonal.

Problem 3. What are the values of $p \in \mathbb{R}$ for which

$$||(x_1, \ldots, x_n)||_p := \sqrt[p]{|x_1|^p + \cdots + |x_n|^p}$$

defines a norm on $\mathbb{C}^n$?

☐ $p \geq 0$ ☐ $p = 2$ ☐ $p \geq 1$ ☐ $p \in \mathbb{N}_+$

Problem 4. What property of a matrix A says that A is Hermitian?

☐ $A = \overline{A}^T$ ☐ $A^2 = Id$ ☐ $A^2 = A$ ☐ $A = (A^{-1})^T$

Problem 5. Let $A \in M_n(\mathbb{R})$ be a projection matrix. What of the following is (always) true?

☐ $\mathrm{tr}(A) = \mathrm{rk}(A)$ ☐ $\mathrm{rk}(A) = n$ ☐ $\det(A) = 1$ ☐ $A^2 = 0$

Problem 6. What is the number of positive permutations of 5 elements?

☐ 20 ☐ 24 ☐ 60 ☐ 120

Problem 7. Which of the following claims is false for $A \in M_n(F)$ and $P \in F[z] = \mathcal{P}(F)$?

- ☐ If $P(A) = 0$, then $P(\lambda) = 0$ for all eigenvalues λ of A.
- ☐ If $P(\lambda) = 0$ for all eigenvalues λ of A, then $P(A) = 0$.
- ☐ $\chi_A(\lambda) = 0$ for all eigenvalues λ of A.
- ☐ $\chi_A(A) = 0$

Problem 8. What is the coefficient in degree $n-1$ of $\chi_A(t)$ for a matrix $A \in M_n(F)$?

☐ $\det A$ ☐ $(-1)^{n-1}\,\mathrm{rk}A$ ☐ $(-1)^{n-1}\,\mathrm{tr}A$ ☐ $(-1)^{n-1}$

Problem 9. What of the following is not an inner product $\langle f, g\rangle$ for $f, g \in C^\omega(\mathbb{R}, \mathbb{R})$?

☐ $\int\limits_{-1}^{1} f'(x)g'(x)\,dx + f(0)g(0)$ ☐ $\int\limits_{-1}^{1} f(x)g(x) \cdot x^2\,dx$

☐ $\sum\limits_{n=0}^{\infty} f^{(n)}(0)g^{(n)}(0) \cdot \dfrac{1}{(3n)!}$ ☐ $\sum\limits_{n=1}^{\infty} f(n)g(n) \cdot \dfrac{1}{n^n}$

Problem 10. How is the property of a norm called which determines whether it comes from an inner product?

☐ positive definiteness ☐ triangle inequality

☐ Cauchy-Schwarz inequality ☐ Parallelogram identity

Problem 11. Which of the below properties does an orthogonal matrix not always have?

- ☐ it is invertible
- ☐ it preserves norm
- ☐ if it is diagonalizable, then its eigenvalues are ± 1
- ☐ any two linearly independent eigenvectors are orthogonal

Problem 12. Assume $A \in M_2(\mathbb{R})$ is a definite matrix, $\mathbf{b} \in \mathbb{R}^2$, and $c > 0$, and the equation $\mathbf{x}^T A\mathbf{x} - 2\mathbf{x}^T\mathbf{b} + c = 0$ determines a regular conic. What of the following types is the conic?

☐ two parallel lines ☐ ellipse ☐ hyperbola ☐ parabola